

Mathletics

Series A, B & C

Problem Solving

Copyright © 2009 3P Learning. All rights reserved.

First edition printed 2009 in Australia.

A catalogue record for this book is available from 3P Learning Ltd.

ISBN 978-1-925202-81-6

Ownership of content The materials in this resource, including without limitation all information, text, graphics, advertisements, names, logos and trade marks (Content) are protected by copyright, trade mark and other intellectual property laws unless expressly indicated otherwise.

You must not modify, copy, reproduce, republish or distribute this Content in any way except as expressly provided for in these General Conditions or with our express prior written consent.

Copyright Copyright in this resource is owned or licensed by us. Other than for the purposes of, and subject to the conditions prescribed under, the Copyright Act 1968 (Cth) and similar legislation which applies in your location, and except as expressly authorised by these General Conditions, you may not in any form or by any means: adapt, reproduce, store, distribute, print, display, perform, publish or create derivative works from any part of this resource; or commercialise any information, products or services obtained from any part of this resource.

Where copyright legislation in a location includes a remunerated scheme to permit educational institutions to copy or print any part of the resource, we will claim for remuneration under that scheme where worksheets are printed or photocopied by teachers for use by students, and where teachers direct students to print or photocopy worksheets for use by students at school. A worksheet is a page of learning, designed for a student to write on using an ink pen or pencil. This may lead to an increase in the fees for educational institutions to participate in the relevant scheme.

Published 3P Learning Ltd

For more copies of this book, contact us at: www.3plearning.com/contact

Designed 3P Learning Ltd

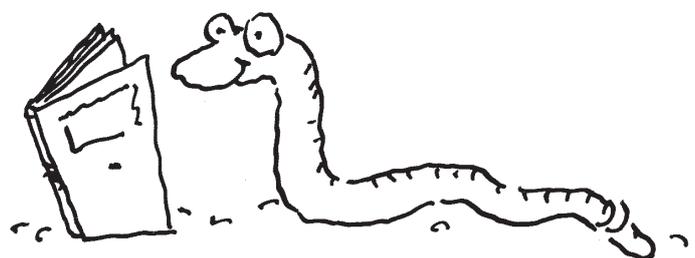
Although every precaution has been taken in the preparation of this book, the publisher and authors assume no responsibility for errors or omissions. Neither is any liability assumed for damages resulting from the use of this information contained herein.

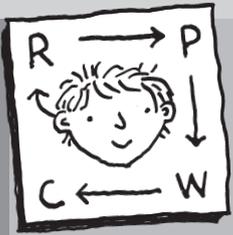
Problem Solving

– Series A, B & C

Contents

Read, plan, work and check	2
Draw a diagram	12
Look for patterns	30
Act it out	40
Trial and error	50
Make a list	60
Estimation	70
Work backwards	80
Open-ended	90





Read, plan, work and check

Rationale

Learning how to structure an investigation is an integral part of developing mathematical thinking. This strategy involves understanding and clarifying the question, selecting and using a strategy to solve the problem, working out a solution and checking the solution in terms of the original question. The use of this strategy can lay a firm foundation in a student's development of problem-solving skills.

.....

Teaching the strategy

- Write a problem appropriate to your students' ages and mathematical knowledge on the board.
eg *There are 6 planes in the sky and 3 planes on the ground.
How many planes altogether?*

One way to solve this problem is to use the *read/plan/work/check* strategy.

Write it on the board and refer to each step as you work through the problem with the students.

1 The first step is to read the problem.

- Have the children read the problem and think about it as they read.
- Discuss the keywords. Underline these words.
- *What do you have to find out? What facts will help you to answer the question? What words will help you know how to solve the problem?*

2 The second step is to make a plan.

- Students think about what they need to do to solve the problem. *Have you seen a similar problem? What is similar about it? What did you need to do? What operation do you need to use?*
- Ask some students to tell the class how they plan to solve the problem.

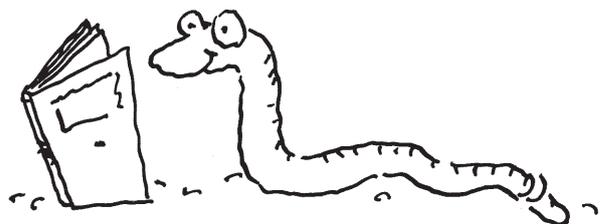
3 The third step is to work out the problem.

- Choose a student to work through the problem. They show any working and write the answer clearly. Ask if any students would solve the problem differently and have them tell the class how.

4 The fourth step is to check the answer. This is when you look back at what you have done.

- Discuss how the answer can be checked to see if it is correct.
- Pose a variety of other problems that allow students to practise this strategy.

Encourage students to find problems in their workbooks and to use them as models to write others for the class or for small groups to solve. Problems can be collected and made into class books to share and can be discussed as a class. These activities will all facilitate the learning of this strategy.



Remind students to clearly show any working out as this will help when checking. Tell them to write answers in correct units, eg o'clock etc. where appropriate. Discuss how you want answers written, either as a number or in a full sentence. When a worksheet is completed ask some students to share their solutions.

Worksheet 1
MIXED PROBLEMS

Six problems: **1, 2** addition; **3, 4** subtraction; **5, 6** addition and subtraction.

Vocabulary: *how many?, how many left?, add, subtract*

Worksheet 3
MIXED PROBLEMS

Four problems: **1** is 3-step addition (answer in km); **2** doubling pounds; **3** addition and change from £20; **4** finding one-quarter and subtraction.

Vocabulary: *altogether, km, cost, how much?, double, add, subtract, how much change?, one-quarter*

Worksheet 5
MIXED PROBLEMS

Four problems: **1** counting in 10s; **2** counting in 5s; **3** multiplying by 10 (answer in minutes); **4** dividing 20 by 2.

Vocabulary: *counting in ten's, how many?, counting in 5s, divide, equally, minutes, multiply*

Worksheet 7
BOOKWORM

One problem involves addition and subtraction.

Vocabulary: *addition, how many more?, subtraction*

Worksheet 2
MIXED PROBLEMS

Six problems: **1** counting on; **2, 5** subtraction; **3** addition (answer in cm); **4** counting on or subtraction (answer in hours); **6** doubling.

Vocabulary: *counting on, how many?, how tall?, how long?, add, subtract, double, hours, cm*

Worksheet 4
MIXED PROBLEMS

Four problems: **1** finding 20-pence pieces; **2** addition and subtraction; **3** addition and seven days in a week; **4** finding how many pence.

Vocabulary: *how many?, costs, altogether, how many more?, are left, pence*

Worksheet 6
MIXED PROBLEMS

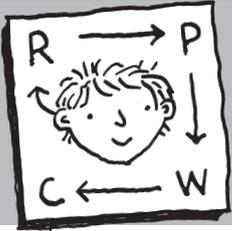
Four problems: **1** counting on; **2** finding 3 coins; **3** addition and subtraction; **4** repeated addition (some students could use multiplication and then addition).

Vocabulary: *counting on, how many?, what coin?, altogether, repeated addition, multiply, add, subtract*

Worksheet 8
DOLLS

One problem with multiple questions: addition and subtraction Question 2 requires students to use the problem as a model to write their own problem and questions.

Vocabulary: *addition, subtraction, how many?, altogether*



Name _____

Date _____

Mixed problems

- 1 4 pencils and 5 pencils.
How many pencils?



- 2 2 green apples and
3 red apples.
How many apples?



- 3 10 cars. 5 drive away.
How many left?



- 4 8 boats. 3 sail away.
How many left?

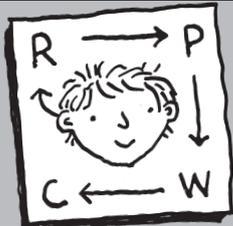


- 5 10 fish. 7 swam away.
4 came back.
How many fish?



- 6 Kaila has 5 white
cats and 4 black cats.
She feeds 7.
How many still
to feed?





Name _____

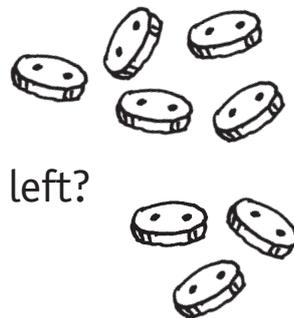
Date _____

Mixed problems

- 1 Roa is 5 years old.
Linn-Linn is 7 years
older than Roa.
How old is Linn-Linn?



- 2 20 buttons.
8 roll away.
How many are left?



- 3 Vi's doll is 20 cm tall.
Jo's doll is 10 cm taller
than Vi's doll.
How tall is Jo's doll?



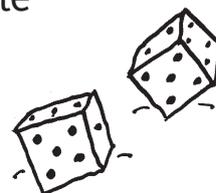
- 4 Erin began reading
at 4 o'clock.
She stopped at
7 o'clock.
How long did she read?

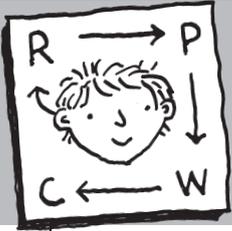


- 5 20 clowns.
15 have hats on.
How many do not
have hats on?



- 6 Skye rolled double
5 on the dice.
What was
her score?





Name _____

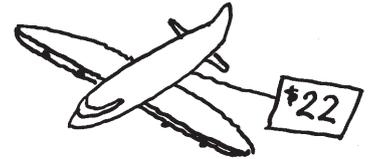
Date _____

Mixed problems

- 1 Kaz ran 5 km on Monday, 10 km on Tuesday and 3 km on Wednesday.
How far did she run altogether?



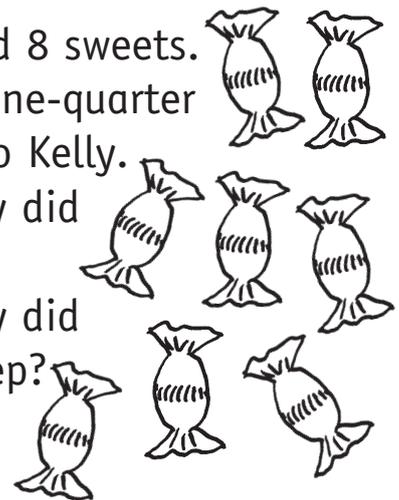
- 2 Dane buys 2 toy planes. Each plane costs £22.
How much does he spend?



- 3 Rebecca bought 2 tubs of ice-cream. They were £4 each.
How much change did she get from £20?

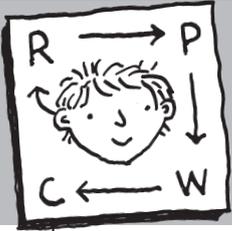


- 4 James had 8 sweets. He gave one-quarter of them to Kelly.
How many did Kelly get?
How many did James keep?



Kelly _____

James _____



Name _____

Date _____

Mixed problems

- 1 Lani has £1.
A flower costs 20 pence.
How many flowers
can she buy?



2



Liz has 16 pencils and Chris
has 3 pencils.

- a** How many pencils
altogether?
b How many more pencils
has Liz than Chris?

a _____

b _____

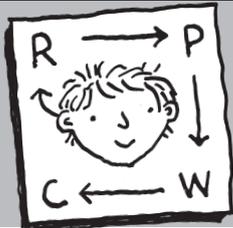
3



Michael borrowed 15 books
from the library.
He reads one every night.
How many books are left to
read after one week?

- 4 Blake has four £1 coins,
five 10 pence coins
and two 5 pence coins.
How many pence does
he have altogether?



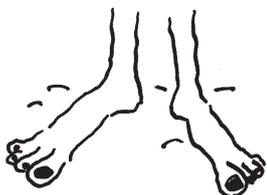


Name _____

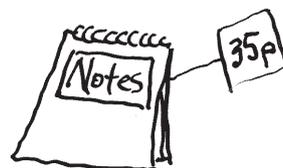
Date _____

Mixed problems

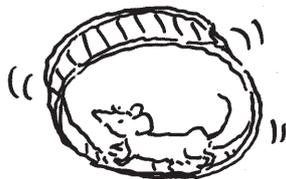
- 1 Tori and her 4 friends painted each other's toenails. How many toenails were painted?



- 2 Wes bought a pad for 35p. How many 5 pence pieces did he need?



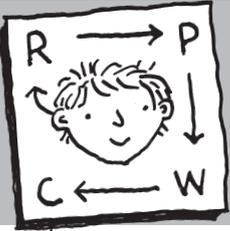
- 3 A mouse runs around the mouse wheel. The wheel turns 10 times every minute. How many turns does the wheel make in 4 minutes?



4



- Bob the butcher has 20 bones. Bob wants to divide the bones equally between 2 dogs. How many bones does he give each dog?



Name _____

Date _____

Mixed problems

1



Brent was on page 93 of his book at 1 o'clock.

At 2 o'clock he was on page 97.
How many pages did he read in one hour?

- 2 Alec wants to buy a goldfish. The goldfish costs 35 pence. Alec uses 3 coins. What coins does he use?



- 3 There are 3 vases. One vase has 4 flowers in it. One vase has 6 flowers in it. There are 15 flowers altogether. How many flowers are in the other vase?



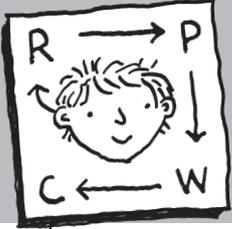
4



On Monday, 3 hens lay 2 eggs each.

On Tuesday, the 3 hens lay 3 eggs each.

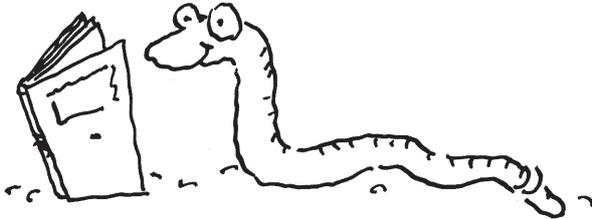
How many eggs altogether?



Name _____

Date _____

Bookworm



On Monday, Bookworm borrowed **20** books from the library.

He read 1 book when he got home.

On Tuesday he read 2 books.

On Wednesday he read 3 books.

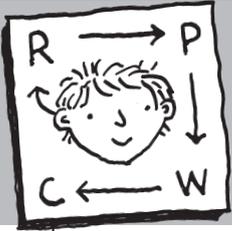
On Thursday he read 2 books and on Friday he read 4 more books.

On Saturday he read another 4 books.

On Sunday he counted the books he had read.

1 How many books had he read? _____

2 How many books did he have to read before he took them all back to the library? _____



Name _____

Date _____

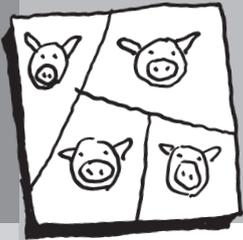
Dolls

Abby has 4 dolls.
 Clare has 6 dolls.
 Katie has 2 dolls.
 Melissa has 3 dolls.



- 1 a** How many dolls do Abby and Clare have altogether? _____
- b** How many dolls do Katie and Melissa have altogether? _____
- c** Clare gives all her dolls to Melissa.
 How many dolls does Melissa now have? _____
- d** Melissa then gives all her dolls to Katie.
 How many dolls does Katie now have? _____
- e** How many dolls does Abby have? _____
- f** How many more dolls does Katie have than Abby? _____

2 Use this problem as a model and write your own problem and questions.



Draw a diagram

Rationale

Visualisations are necessary for students' understanding of mathematical concepts and relationships. Drawing a picture or diagram can help students solve problems as it allows them to visualise a problem more clearly. It helps them to solve more difficult problems.



Teaching the strategy

- Write a problem appropriate to your students' ages and mathematical knowledge on the board.
eg 20 balloons, 6 fly away. How many left?
- *One way we can solve this problem is by doing a drawing.* Tell the students that drawings need not be elaborate and they should only contain enough detail to solve the problem.
- Tell students that even strokes can be used to represent objects. Draw 20 strokes on the board and say, *These are the 20 balloons.*
- Cross 6 out. *These are the balloons that have flown away.*
- *We can see how many balloons are left. We count 14, so there are 14 balloons left.*
- *How can we check to see if we are right? We count one ... fourteen and then add the six that flew away. The answer is 20 so we are correct.*
- *We can then show what we have done*
 $20 - 6 = 14.$
- Pose other simple problems and ask students to do drawings on the board to solve them.
- For further practice, students look for suitable problems in their workbooks and draw diagrams to solve them. Encourage students to write their own problems to share with others. Class books can be made. As this strategy is very visual, quite difficult problems can be posed to the class.
eg A dog has 30 fleas on each leg. How many fleas does he have altogether?

As students complete worksheets ask them to share their solutions with the rest of the class.



Reinforce that students should read each question carefully before starting. Remind them that drawings do not have to be elaborate. Discuss how you want them to write the answers. Finally remind them to check their work. Stress that for each worksheet a drawing or diagram will be the means of solving the problem.

Worksheet 1
AT THE ZOO

Vocabulary: *how many altogether?, lots of, count*

Worksheet 3
AT THE BUS STOP and MITCH'S TOWERS

Vocabulary: *how many?, add, subtract, take away, cross out, total, count*

Worksheet 5
MARBLES and PENS

Vocabulary: *more, how many?, add, take away, cross out, lots of, count*

Worksheet 7
CROCODILE EGGS and JULIAN'S GARDEN

Vocabulary: *lots of, take away, subtract, cross out, how many?, rows, count*

Worksheet 9
AMY'S BIRTHDAY CANDLES

Students draw to answer parts **a** and **b** and use the information to answer **c**. **Vocabulary:** *how many?, how many more?, add, subtract, count*

Worksheet 11
NATE'S BIRTHDAY and MR SNAIL

Vocabulary: *rows, how many?, metres, up, down, how long? add, subtract, lots of, count*

Worksheet 13
THE CAR PARK

Vocabulary: *counted, how many?, lots of 4, left, subtract, take away, add, count*

Worksheet 15
MR FROST'S FLATS

Vocabulary: *odd, even, lots of, how many?, add, count*

Worksheet 2
JANE'S CUPCAKES

Model how to be systematic when drawing the Smarties on the cakes.

Vocabulary: *how many?, share equally, count*

Worksheet 4
ASHA'S CARS and PENCILS

Vocabulary: *have left?, how many?, multiply, take away, subtract, equal piles, count*

Worksheet 6
TEDDY RACE and THE SPACEMEN

For *Teddy race* suggest that students use coloured pencils.

Vocabulary: *first, last, behind, after, which? 2nd, how many?, count*

Worksheet 8
THE FISH and THE STAIRS

For *The fish* students draw the fish in stages.

Vocabulary: *long, half, length, add, how long?, bottom, 2nd, middle, four more up, centimetres, add, take away*

Worksheet 10
GRANDMA'S PRESENT and ANDY'S POOL

Vocabulary: *rows, how many?, each, cost £1, how much?, square, side, 6 m long, metres, count*

Worksheet 12
ON THE FARM

Vocabulary: *how many?, add, lots of, total, count*

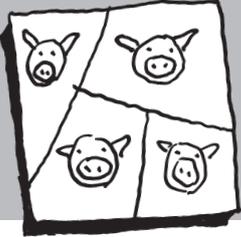
Worksheet 14
FREDDY FROG

When the problems are solved the class could discuss any patterns found. **Vocabulary:** *how many?, lots of, counting in 2s, 4s, 5s, 10s*

Worksheet 16
SALLY THE CUPCAKE QUEEN

When the problems are solved the class could discuss any patterns found.

Vocabulary: *how many?, 2nd, 3rd, 4th, count*



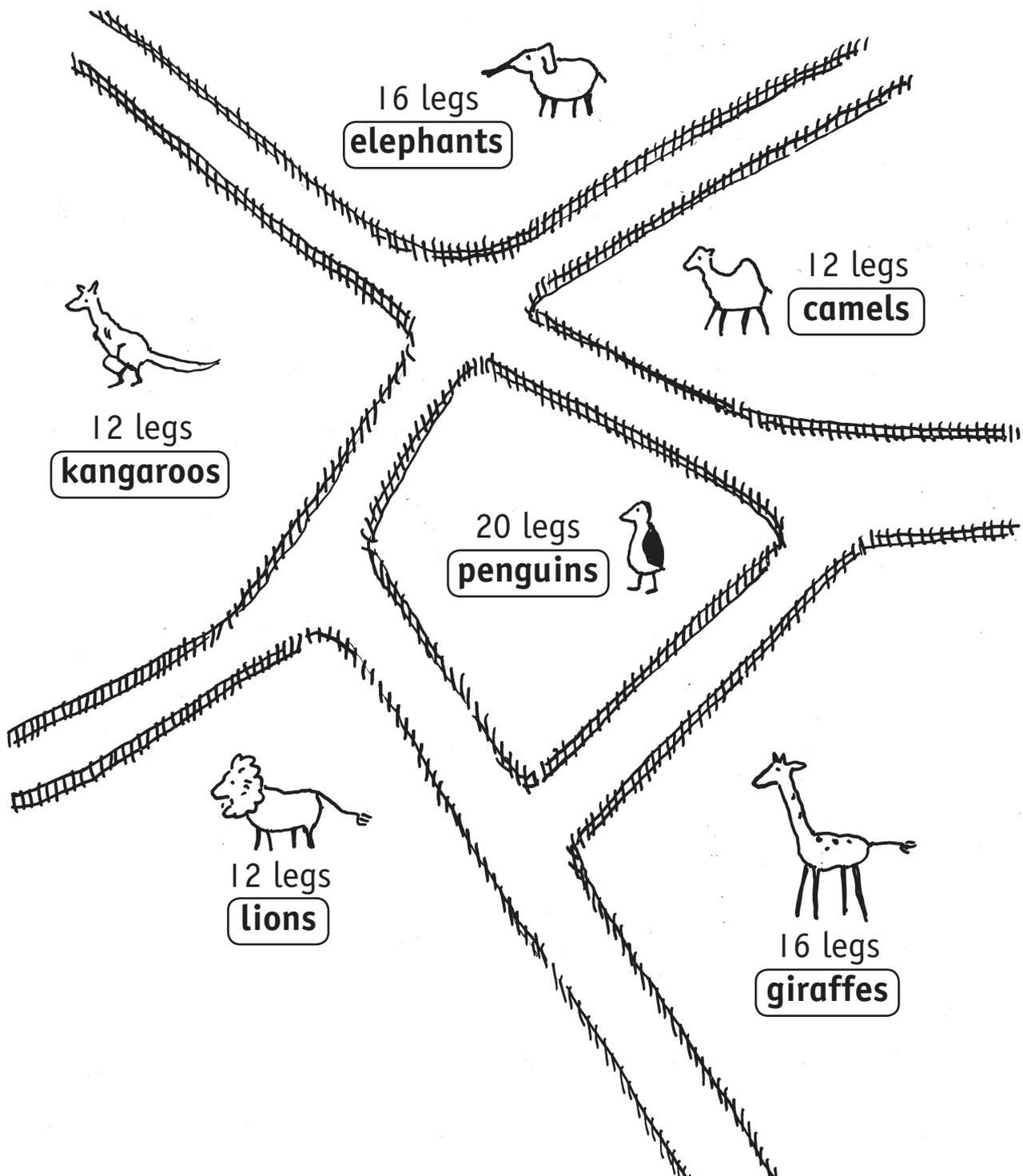
Name _____

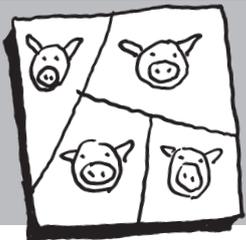
Date _____

At the zoo

Jen went to the zoo. She counted legs.

How many animals did she see altogether? _____





Name _____

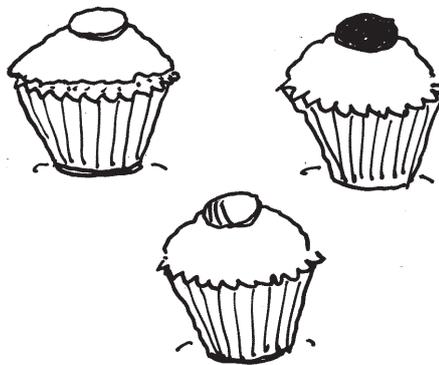
Date _____

Jane's cupcakes

Jane made six cupcakes.

She iced them and then put Smarties on them.

How many Smarties could she put on each cupcake if she had:

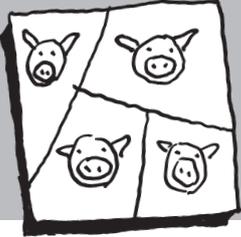


1 6 Smarties? _____

2 12 Smarties? _____

3 18 Smarties? _____

4 24 Smarties? _____



Name _____

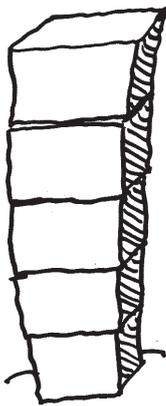
Date _____

At the bus stop

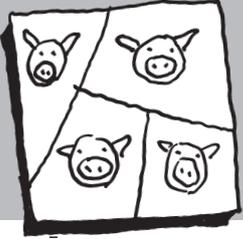
There are 9 girls and 5 boys on a bus.
 At the first stop, 3 children get off.
 At the next stop, 4 children get off.
 How many children are left on the bus?



Mitch's towers



Mitch had 20 blocks. He made 4 towers.
 One tower had 3 blocks.
 One tower had 5 blocks.
 One tower had 2 blocks.
 How many blocks were in the other tower?

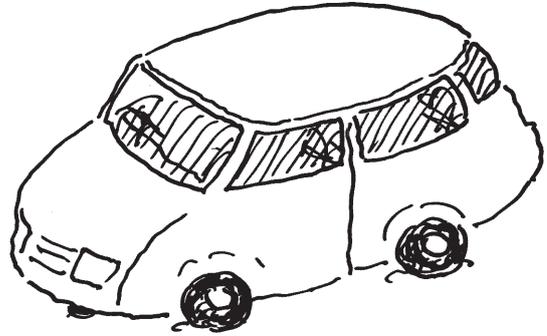


Name _____

Date _____

Asha's cars

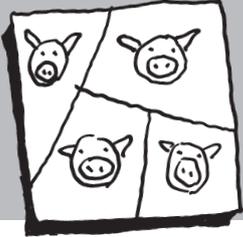
Asha has 18 toy cars.
He gives his 6 friends 2 cars each.
How many cars does he have left?



Pencils



Pam has 18 pencils.
She puts them into 3 equal piles.
How many pencils are in each pile?

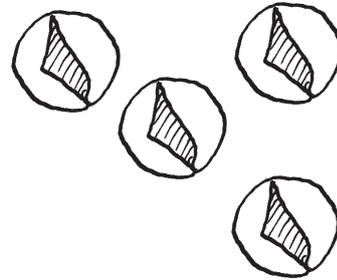


Name _____

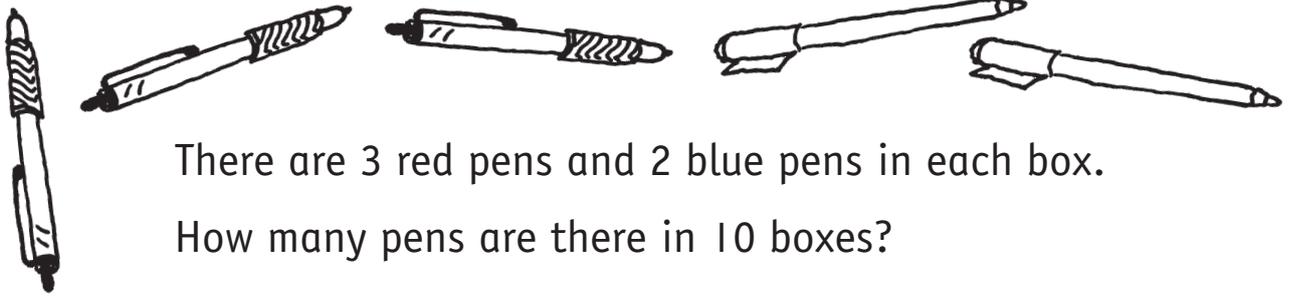
Date _____

Marbles

Todd bought 8 marbles.
 Bob bought 2 more than Todd.
 Bob then lost 3.
 How many marbles does Bob have now?



Pens

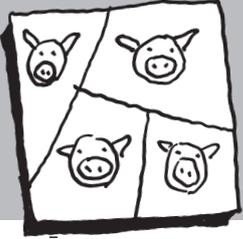


There are 3 red pens and 2 blue pens in each box.

How many pens are there in 10 boxes?

How many red pens are there in 10 boxes?

How many blue pens are there in 10 boxes?



Name _____

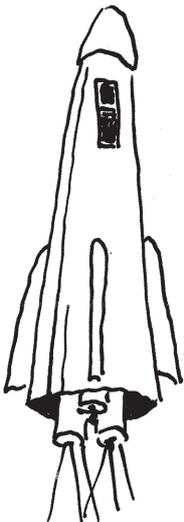
Date _____

Teddy race

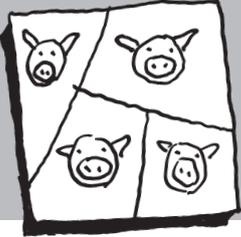
Five teddies were in a race.
 Red teddy came first.
 Green teddy came last.
 Orange teddy came behind blue teddy.
 Yellow teddy came after orange teddy.
 Which teddy came 2nd?



The spacemen



6 spacemen fit in each spaceship.
 4 spaceships are up in space.
 How many spacemen are up in space?

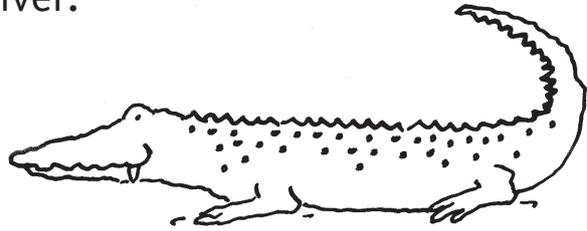


Name _____

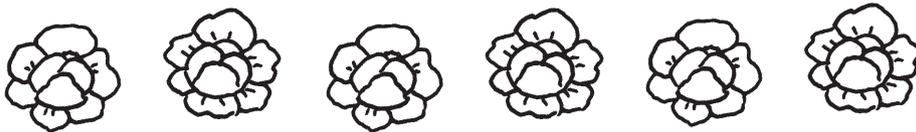
Date _____

Crocodile eggs

10 crocodiles went down to the river.
 5 of them laid eggs.
 They laid 4 eggs each.
 A goanna ate 12 of the eggs.
 How many eggs were left?



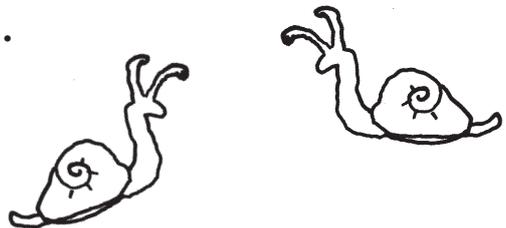
Julian's garden

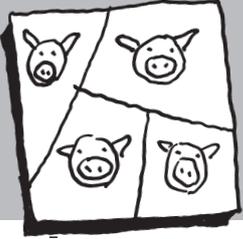


Julian planted 6 rows of lettuce.
 He planted 4 heads of lettuce
 in each row.

Snails ate 4 heads of lettuce.

How many heads of lettuce were left?



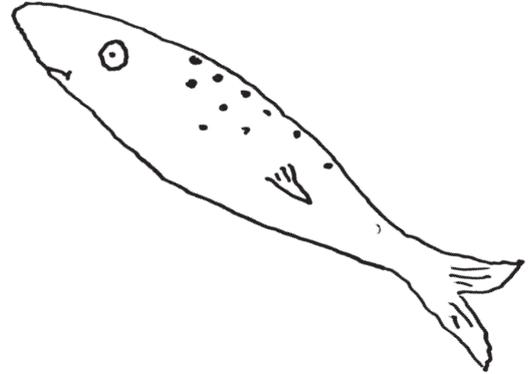


Name _____

Date _____

The fish

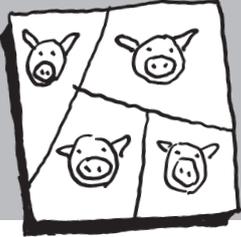
The head of the fish is 6 cm long.
 The tail is half the length of the head.
 The body is as long as the head and the tail together.
 How long is the fish?



The stairs



Jim is at the bottom of the stairs
 Tom is on the second stair.
 Sandy is on the middle stair.
 Suri is at the top. She is five stairs
 higher than Tom.
 What stair is Sandy on?

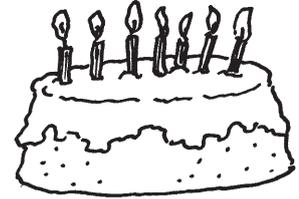


Name _____

Date _____

Amy's birthday candles

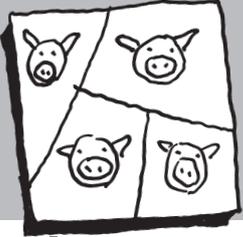
- 1 Amy is 7 years old today.
How many birthday candles has she blown out since she was born? _____



- 2 Amy's brother is 3 years old.
How many birthday candles has he blown out since he was born? _____



- 3 How many more candles has Amy blown out than her brother? _____

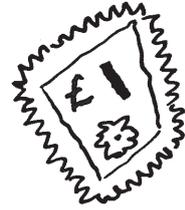


Name _____

Date _____

Grandma's present

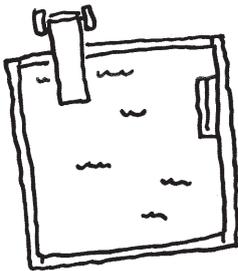
Alice sent a birthday present to her grandma. She bought some stamps and put them on the parcel. There were 3 rows of stamps and there were 5 stamps in every row.



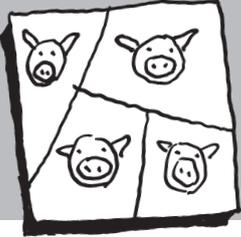
1 How many stamps were there? _____

2 If each stamp cost £1, how much did it cost to send the parcel? _____

Andy's pool



Andy had to fence his swimming pool. The pool was a square with sides 6 metres long. Andy put posts every 3 metres. How many posts did he need?

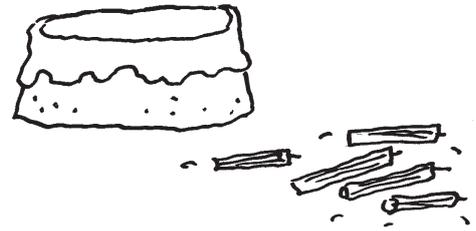


Name _____

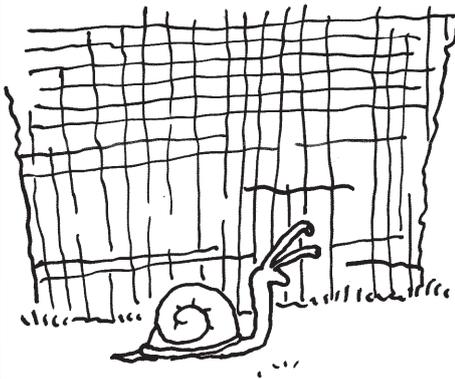
Date _____

Nate's birthday

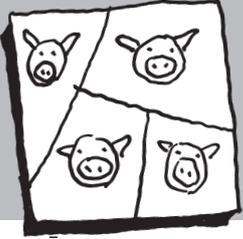
It was Nate's birthday.
 Nate put candles on the cake.
 He put 3 rows with 4 candles in each row.
 Everyone sang Happy Birthday.
 Nate blew out 8 candles.
 How many candles were not blown out?



Mr Snail



Mr Snail had to climb a wall.
 The wall was 9m high.
 Mr Snail climbed 3m up the wall every day.
 Every night he would slip 1m down the wall.
 How long will it take Mr Snail to climb
 the wall?



Name _____

Date _____

On the farm

On the farm are:

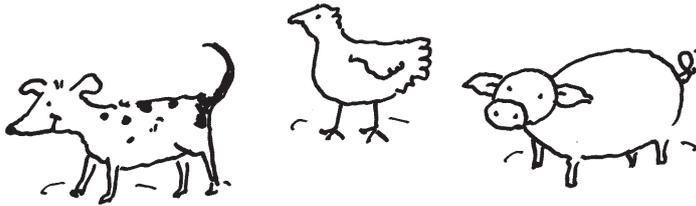
6 horses,

10 cows,

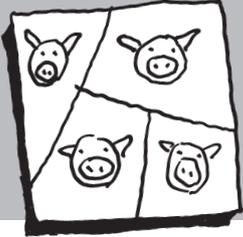
2 dogs,

4 hens

and 5 pigs.



- 1 How many eyes are on the farm? _____
- 2 How many tails are on the farm? _____
- 3 How many legs are on the farm? _____
- 4 How many heads are on the farm? _____

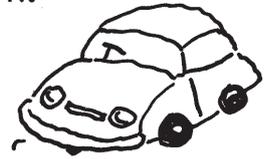


Name _____

Date _____

The car park

- 1 At 7 o'clock, Brendan counted 5 cars in the car park.
How many wheels are in the car park?



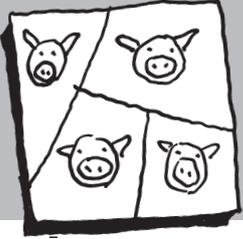
- 2 At 9 o'clock, Brendan counted 44 wheels in the car park.
How many cars are there in the car park now?



- 3 At 5 o'clock in the afternoon, Brendan counted 24 wheels
in the car park.

a How many cars are in the car park? _____

b How many cars left the car park? _____



Name _____

Date _____

Freddy Frog



Freddy Frog liked jumping across the pond.
There were 20 lily pads in the pond.



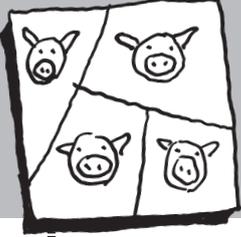
How many jumps would he take to cross the pond if he jumped:

1 2 lily pads at a time? _____

2 4 lily pads at a time? _____

3 5 lily pads at a time? _____

4 10 lily pads at a time? _____

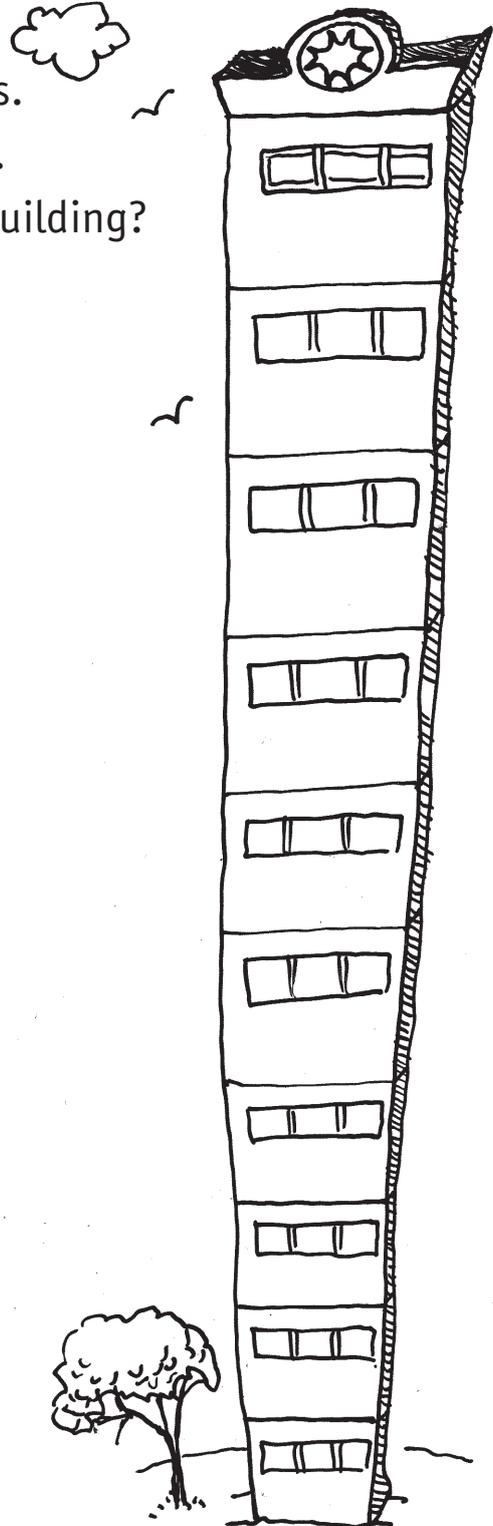


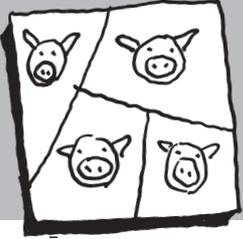
Name _____

Date _____

Mr. Frost's flats

Mr Frost's building is 10 floors high.
All the even numbered floors have 2 flats.
All the odd numbered floors have 3 flats.
How many flats are there in Mr. Frost's building?





Name _____

Date _____

Sally the cupcake queen

Sally made 20 cupcakes.

She put them in a line.

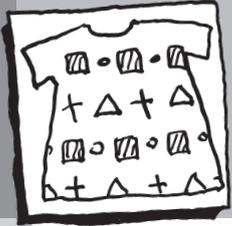
She put pink icing on every 2nd cupcake.

She put a red jelly bean on every 3rd cupcake.

Then she put a yellow Smartie on every 4th cupcake.



- 1 How many cupcakes had pink icing? _____
- 2 How many cupcakes had a jelly bean? _____
- 3 How many cupcakes had a Smartie? _____
- 4 How many cupcakes had icing and a jelly bean? _____
- 5 How many cupcakes had icing, a jelly bean and a Smartie on them? _____
- 6 How many cupcakes had nothing on them? _____



Look for patterns

Rationale

Look for patterns is a very important strategy for problem solving. By observing each given element, one at a time in consecutive sequence, students can decide what the next elements will be in the pattern. Predictions based on these patterns can be used to solve many different kinds of problems. The development of these skills in young students lays a foundation for later algebra work.



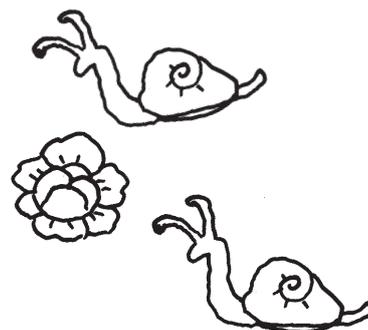
Teaching the strategy

In these early stages of mathematics learning, students should be given many opportunities to practise the following as they will all help with the *Look for patterns* strategy.

- Recognise, copy, continue and create repeating patterns using shapes, objects and pictures.
- Describe a repeating pattern made from shapes.
- Describe a repeating pattern in terms of a number, eg 00^*00^* is a 'three pattern'.
- Recognise, copy, continue and describe simple number patterns that increase and decrease, eg 2, 4, 6, 8 or 10, 9, 8, 7.
- Identify and describe patterns when counting forwards or backwards by 1s, 2s, 5s or 10s.
- Represent number patterns on a hundred chart.
- Determine missing elements in a number pattern.
- Create different patterns and describe the rule.

To model this strategy:

- Write a sample problem appropriate to your students' ages and mathematical knowledge on the board.
eg Luke has written a number pattern that begins with 2, 4, 6, 8, 10.
If he continues this pattern, what are the next four numbers?
- *What do you need to find? You need to find four numbers after 10.*
- *We can solve this problem by finding a pattern. Look at the numbers. The next number depends on the number before it.*
- *What is the rule? It is + 2.*
- *So what are the next four numbers? Write the numbers on the board as students suggest them.*
- *Have we answered the problem? Yes we have.*



Students will be creating or continuing patterns for these problems. When they complete a worksheet have them share their solutions with the class. With **Worksheets 1** and **2** ask some students to describe the pattern they have drawn. With **Worksheets 5** and **6** ask some students to share the problems they have written. Give students time to solve some of these problems.

Worksheet 1
DI'S BEADS

Some students describe their patterns.

Vocabulary: *pattern, how many?, repeat*

Worksheet 2
DARREN'S TOY CARS

Some students describe their patterns.

Vocabulary: *pattern, repeat*

Worksheet 3
ACROBATS

Students identify and continue a pattern and give the rule.

Vocabulary: *number pattern, what is the rule?*

Worksheet 4
BAKED BEANS

Students identify and continue a pattern and give the rule.

Vocabulary: *number pattern, what is the rule?*

Worksheet 5
HUNGRY SNAILS

Students recognise and continue a simple number pattern that increases and give the rule.

Vocabulary: *pattern, what is the rule?*

Worksheet 6
I CAN JUMP

Students count in twos and fives to 18. They write a problem to be shared with the class.

Vocabulary: *pattern, how many jumps?*

Worksheet 7
THE MAGIC PLANT

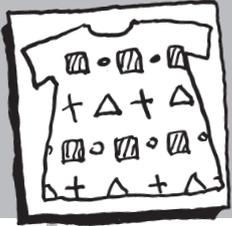
Students discuss the number pattern found.

Vocabulary: *how many? pattern*

Worksheet 8
LETTER DROP

Students fill in missing numbers on the hundred chart. Suggest that students use three different colours to mark the patterns. As a challenge students could find which houses would receive letters if they were delivered to every 3rd,

Vocabulary: *2nd, 3rd, 5th, 100, pattern*



Name _____

Date _____

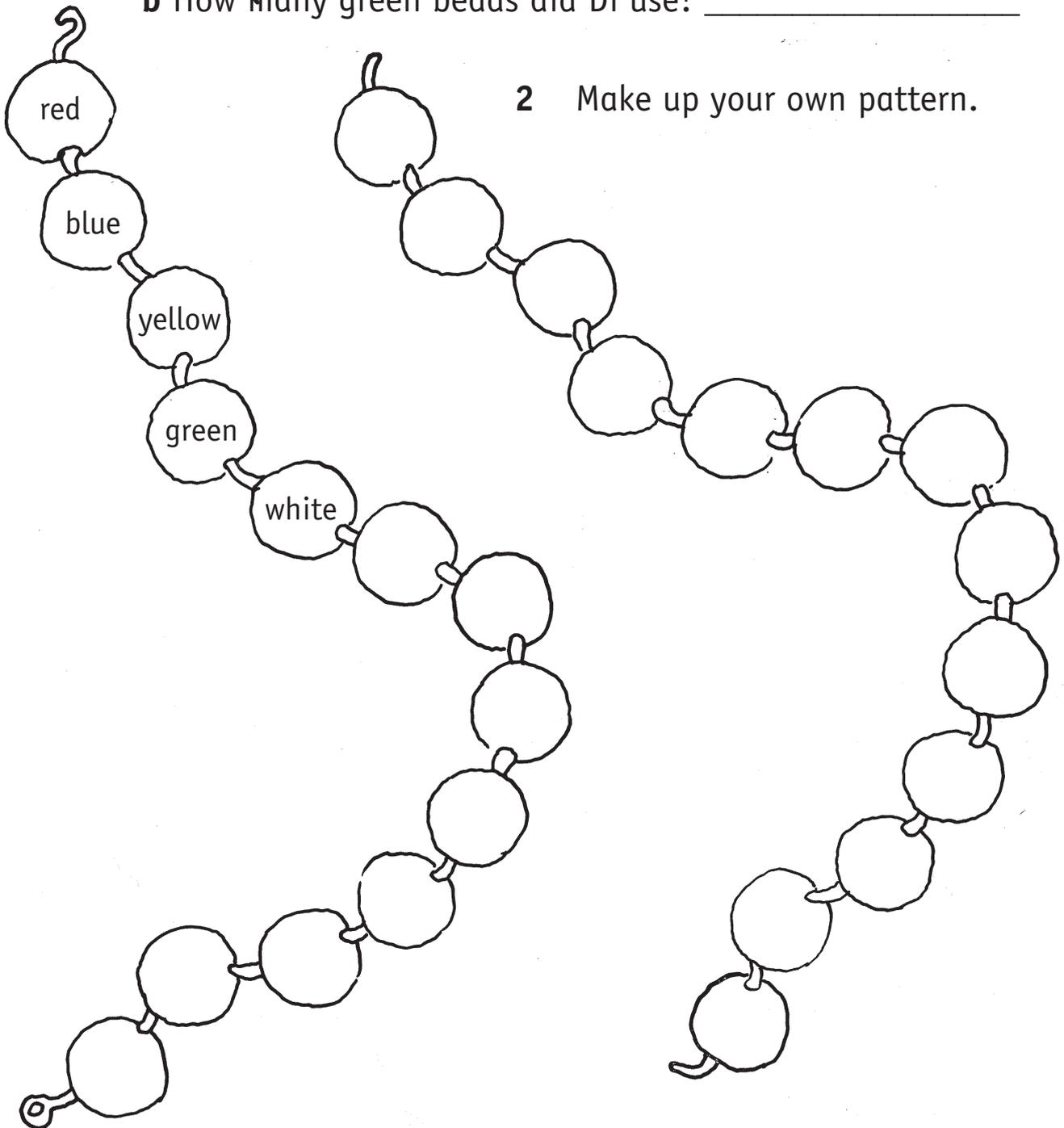
Di's beads

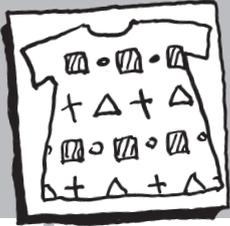
1 Di made a necklace.
She followed a red, blue, yellow, green, white pattern.
Colour the necklace.

a How many red beads did Di use? _____

b How many green beads did Di use? _____

2 Make up your own pattern.





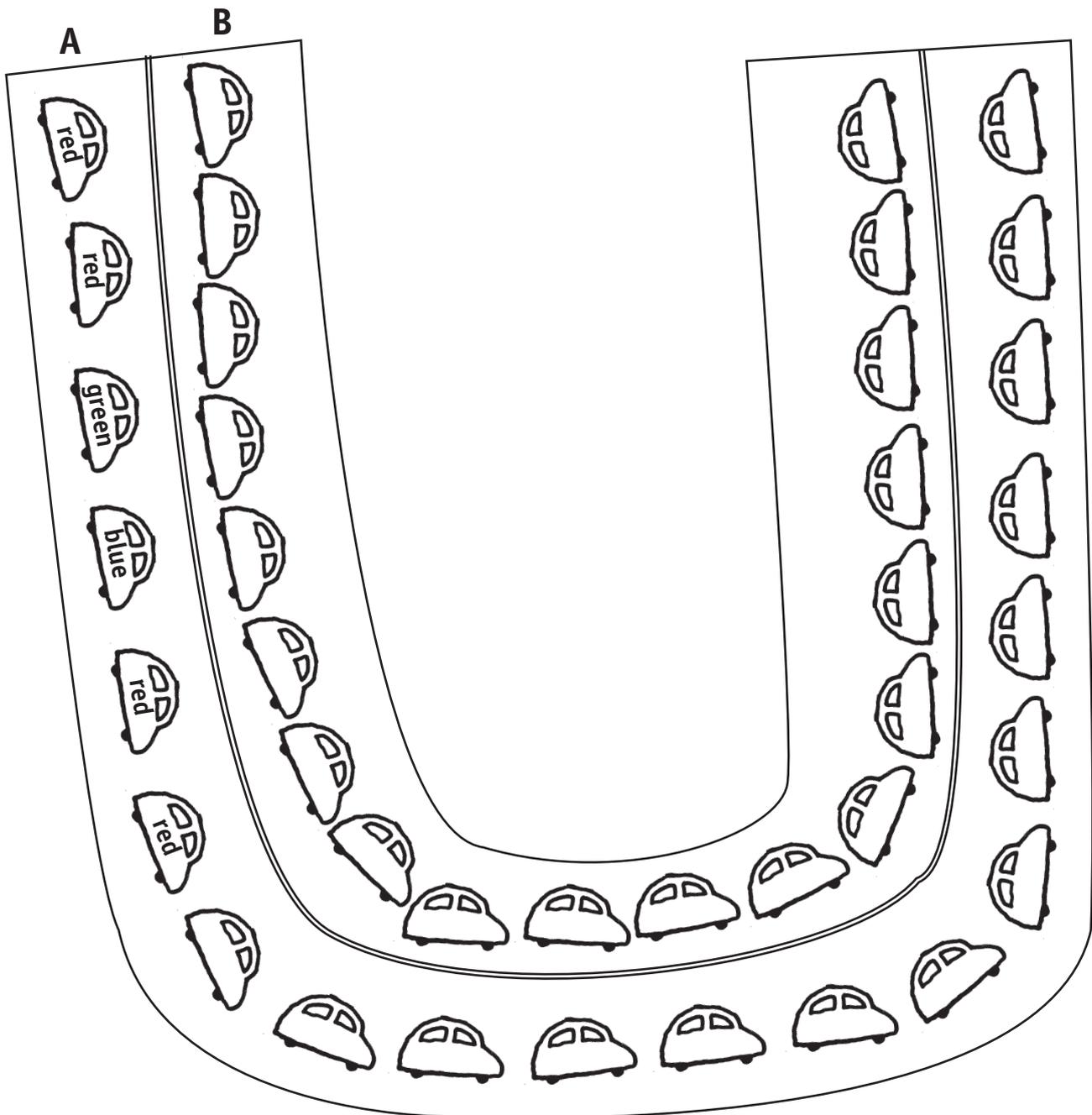
Name _____

Date _____

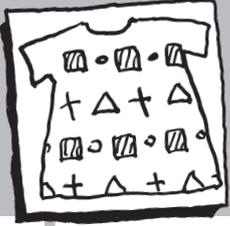
Darren's toy cars

- 1 Darren has 20 cars. They are red, green or blue.
Darren lines them up on Track A.

What colour is the last car? _____



- 2 Make up a pattern on Track B.



Name _____

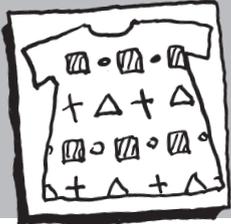
Date _____

Acrobats

1 Follow the pattern and draw more acrobats.



2 What is the rule? _____



Name _____

Date _____

Baked beans

Help Tom pack the shelves.
Follow the pattern and draw the tins.



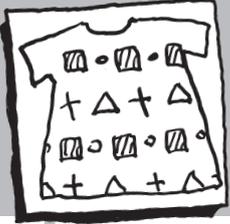
1 What is the rule? _____



2 What is the rule? _____



3 What is the rule? _____



Name _____

Date _____

Hungry snails

1 Ben planted 20 heads of lettuce.

Each night snails ate some of his lettuce.

On Monday there were **18** heads of lettuce left.

On Tuesday there were **16** heads of lettuce left.

On Wednesday there were **14** heads of lettuce left.

On Thursday there were _____ heads of lettuce left.

On Friday there were _____ heads of lettuce left.

On Saturday there were _____ heads of lettuce left.

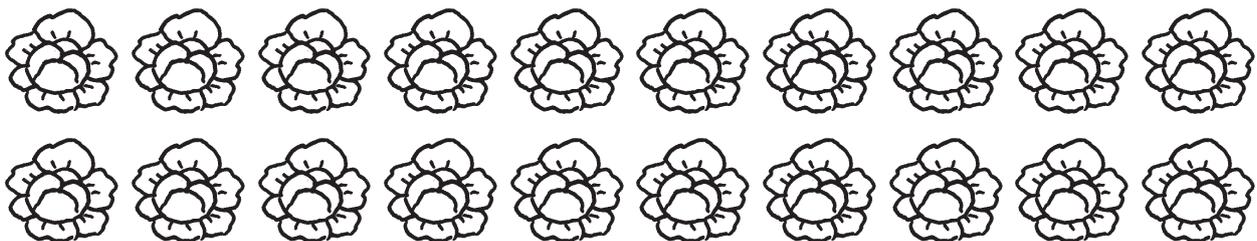
On Sunday there were _____ heads of lettuce left.

On _____ there were _____ heads of lettuce left.

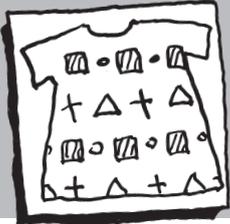
On _____ there were _____ heads of lettuce left.

On _____ there were _____ heads of lettuce left.

What is the rule? _____



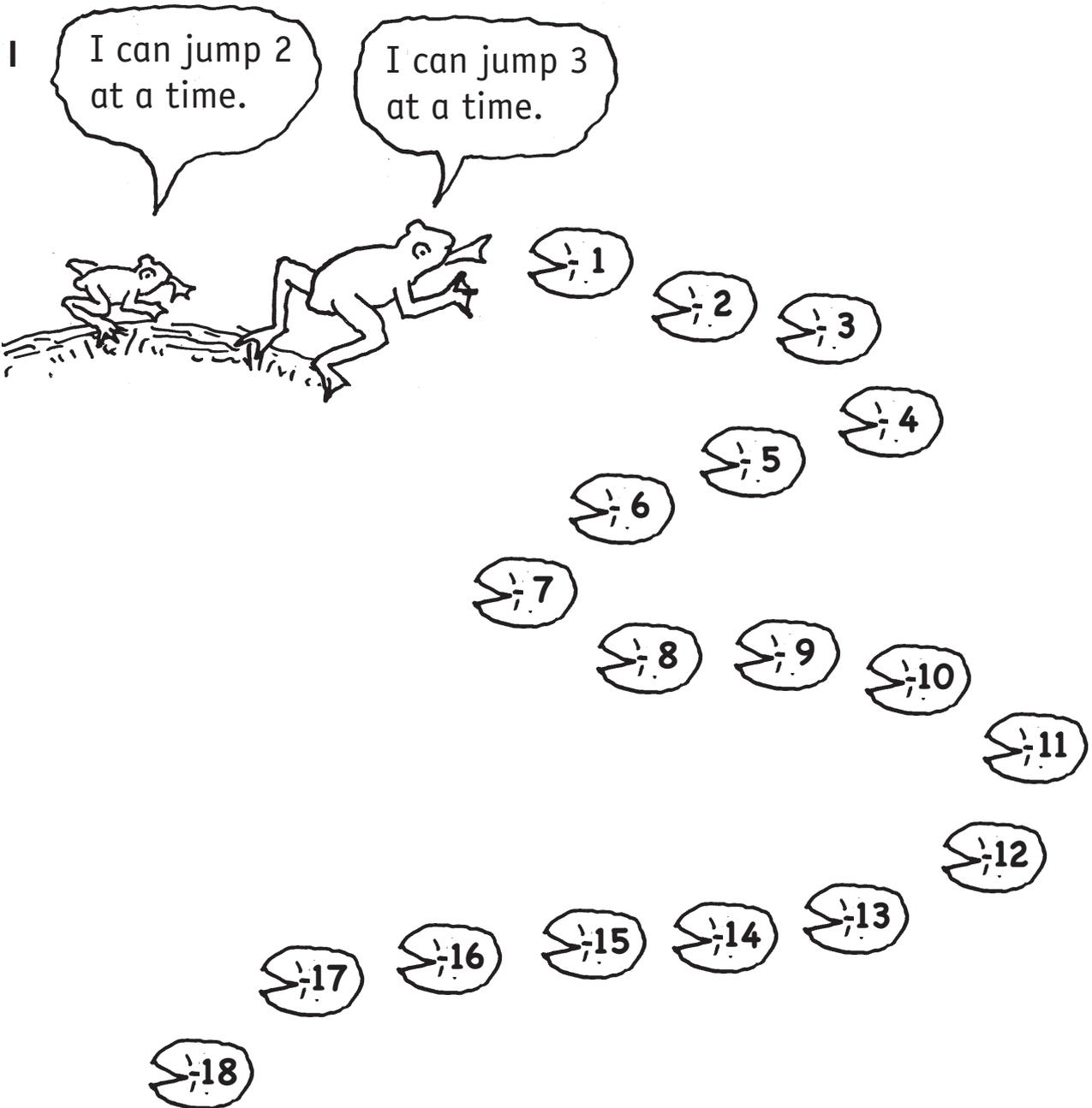
2 Write a problem for a friend.



Name _____

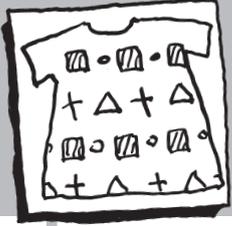
Date _____

I can jump



- a How many jumps does the big frog take to cross the pond? _____
- b How many jumps does the little frog take to cross the pond? _____
- c Which lily pads do they both land on? _____

2 Write a problem for a friend.



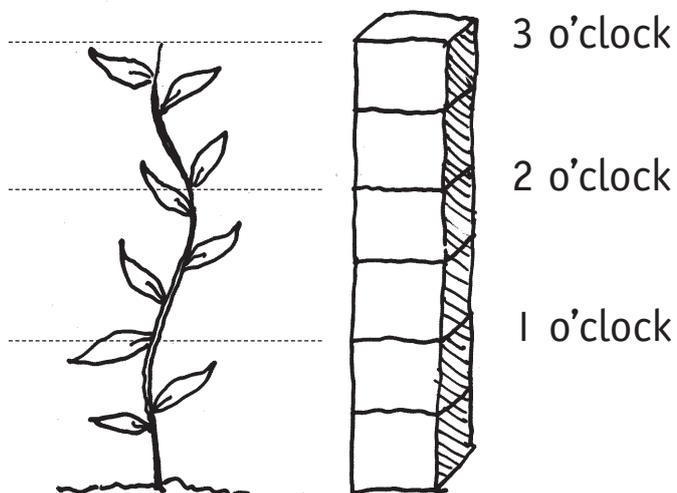
Name _____

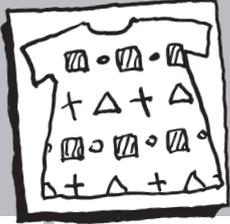
Date _____

The magic plant

Simon was given a magic plant.
It grew very quickly.
Simon used blocks to measure it.

- 1 How many blocks tall will it be at 5 o'clock? _____
- 2 How many blocks tall will it be at 6 o'clock? _____





Name _____

Date _____

Letter drop

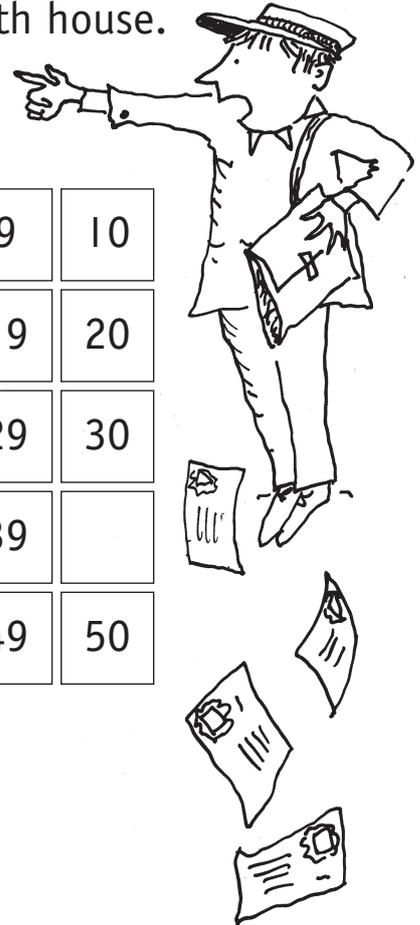
On Monday, Postman Bob delivered a letter to every 2nd house.

On Tuesday he delivered a letter to every 3rd house.

On Wednesday he delivered a letter to every 5th house.

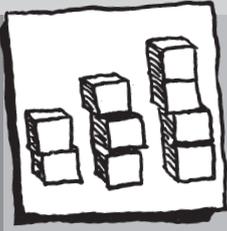
1 Fill in the missing numbers first.

1	2	3	4	5	6		8	9	10
11	12	13		15	16	17	18	19	20
	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	
41	42	43	44		46	47	48	49	50



2 Which houses got no letters?

3 Which houses got 1 letter?



Act it out

Rationale

The importance of the use of this strategy to help students form mathematical concepts is well established. Using concrete materials provides a foundation of practical experience on which students can build abstract ideas. It encourages them to be inventive, aids concept and skill development and enhances understanding of processes. It helps clarify ideas, develop confidence and it encourages independence. It is an effective strategy for those who have difficulty visualising a problem. Acting out a problem can greatly simplify finding solutions.

.....

Teaching the strategy

- Write a sample problem appropriate to your students' ages and mathematical knowledge on the board.
*eg Jamie, the cook, is decorating five cakes.
He puts 4 Smarties on top of each cake.
How many Smarties does Jamie need?*
- Have students read the problem. *What have you been asked to find?*
- *What information have you been given?*
- *One way to solve the problem is to act out the problem. We can use people or objects.*
- *We can use children to be the cakes. Choose five children to be the cakes. We can use counters for the Smarties. Have a child give each 'cake' 4 counters to represent the Smarties.*
- *Let's count the Smarties together.*
- *How many Smarties does Jamie need?*
- Have students suggest other objects that could have been used to represent the cakes and Smarties.

Discuss that any object can be used to represent the situation the students are trying to solve. Students could use people or objects exactly as described, or they could use items, such as blocks etc.

When students are using the worksheets tell them they can choose whatever equipment they think best. In the more difficult worksheets, remind students to record their working clearly. This helps when they need to check answers. As they complete worksheets have them share solutions with the class and discuss what objects were used and how solutions were reached.

Encourage students to write their own problems to share with other students. Class books can be made. Some of the more able students could even write and solve quite difficult problems as this strategy is so visual. These problems can provide models for other problems.

Have students suggest other problems and discuss them as a class.



Before working each sheet, remind students that they can select any object to help with the solution. Allow them to ask for suggestions if they are unsure.

Worksheet 1
BUTTONS, BALLOONS, CARS
3 problems

Vocabulary: *how many? rows, add, lots of*

Worksheet 3
TRAINS

Vocabulary: *how many?, longest, shortest, different*

Worksheet 5
WHO SITS WHERE?

Students can work in groups of nine or ten. Nine students take the name of a child from the problem and they use the clues to find out where each child sits. Remind students to record the solution before they move.

Vocabulary: *between, in front of, behind*

Worksheet 7
MILK BOTTLES

Egg cartons can be divided into 3 and given to students to represent the crates. Remind students to record each way as they go and check to see that ways are different.

Vocabulary: *how many?, two, different*

Worksheet 2
CHICKEN POX

Students could work in pairs. Discuss with students any number patterns that were found. Later this problem could be visually depicted using stick on spots on drawn octopi and displayed.

Vocabulary: *lots of, how many? pattern*

Worksheet 4
FARMER BROWN'S ANIMALS

Vocabulary: *twice, how many?, shared equally*

Worksheet 6
TEDDY TOWN

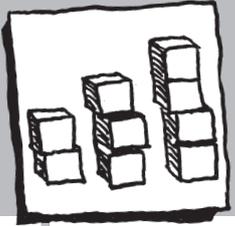
Discuss why all students will not have the same solution.

Vocabulary: *row, column*

Worksheet 8
WHO IS LEFT?

Students can work in groups of 10 and can represent the children in the worksheet. Remind them to record as they go and to stay in their spots until the problem is solved. Discuss if they found any patterns.

Vocabulary: *counting, ordinals 1st ... 10th*



Name _____

Date _____

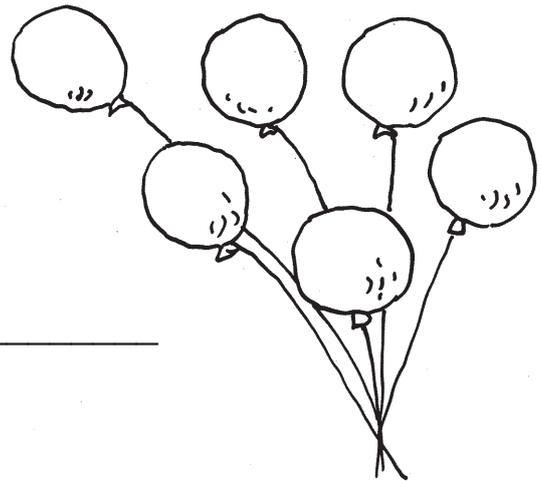
Buttons

Mother bear made her 5 bear cubs new coats.
The coats have 3 buttons each.
How many buttons does mother bear need? _____



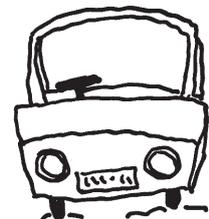
Balloons

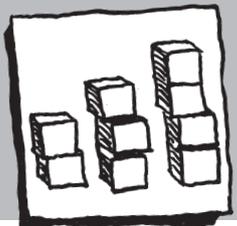
Paula had 6 red balloons.
She blew up some more balloons –
6 blue, 5 yellow, 4 purple, 9 orange,
6 black and 2 white.
How many balloons did Paula have? _____



Cars

In the car park, the cars were parked 7 in a row.
There were 7 rows of cars.
How many cars were in the car park? _____





Name _____

Date _____

Chicken pox

Otti, the octopus, got the chicken pox.

Otti came out in spots on his legs.

1 How many spots on each leg if he had:

a 8 spots _____

b 32 spots _____

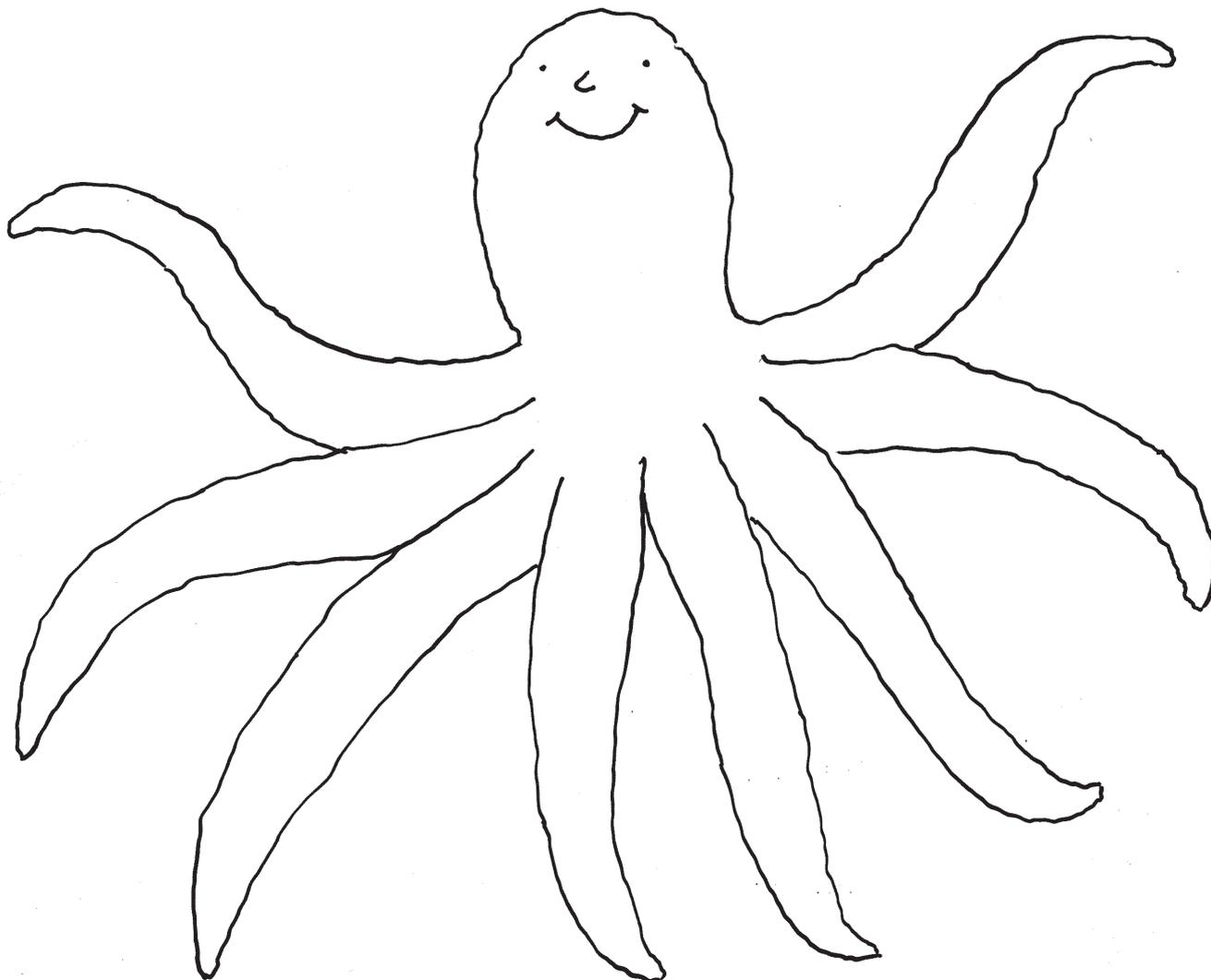
c 64 spots _____

d 80 spots _____

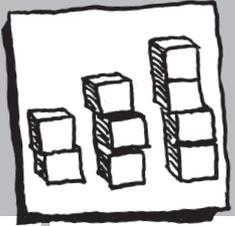
e 112 spots _____

f 160 spots _____

(You have to put the same number on each leg every time.)



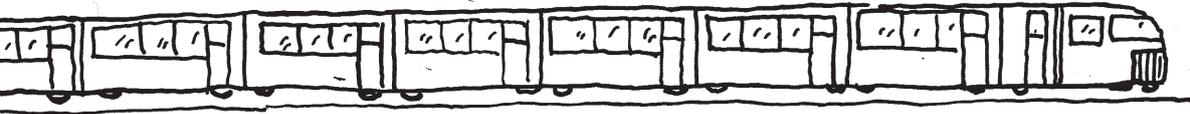
2 Write a problem for a friend.



Name _____

Date _____

Trains



There are 30 passenger cars.

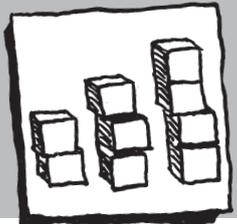
They are going to be put together to make up some trains.

The longest train has 10 passenger cars.

The shortest train has 2 passenger cars.

1 How many different trains can you make?

2 Draw 4 of the trains.

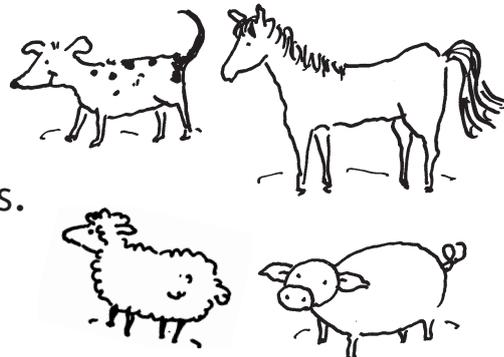


Name _____

Date _____

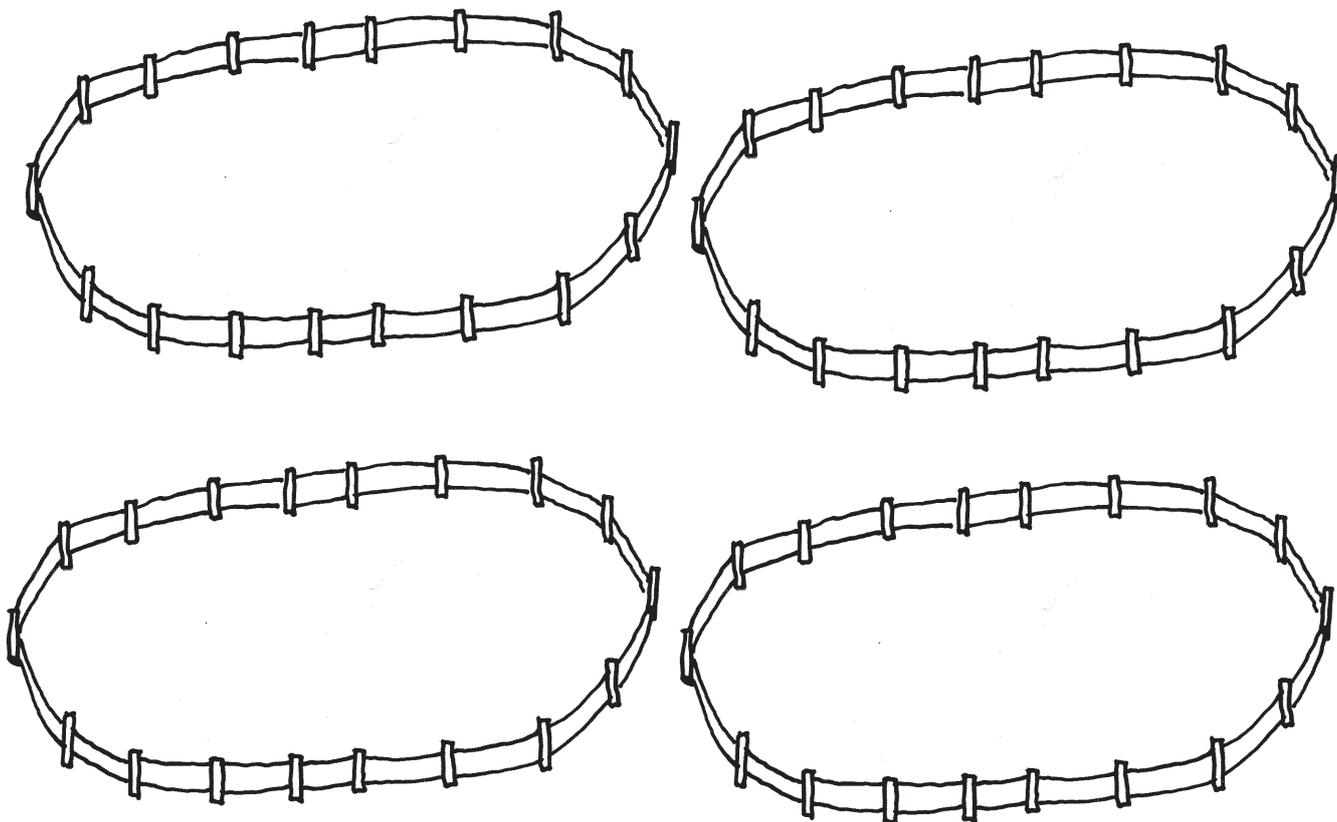
Farmer Brown's animals

- 1 There are 4 dogs on the farm.
 There are twice as many pigs as dogs.
 There are twice as many horses as pigs.
 There are 4 more sheep than horses.
 How many of each animal?



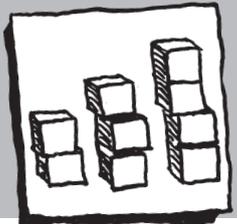
dogs _____; pigs _____; horses _____; sheep _____

- 2 All the animals are shared equally between the paddocks.



- 3 How many of each animal are there in a paddock?

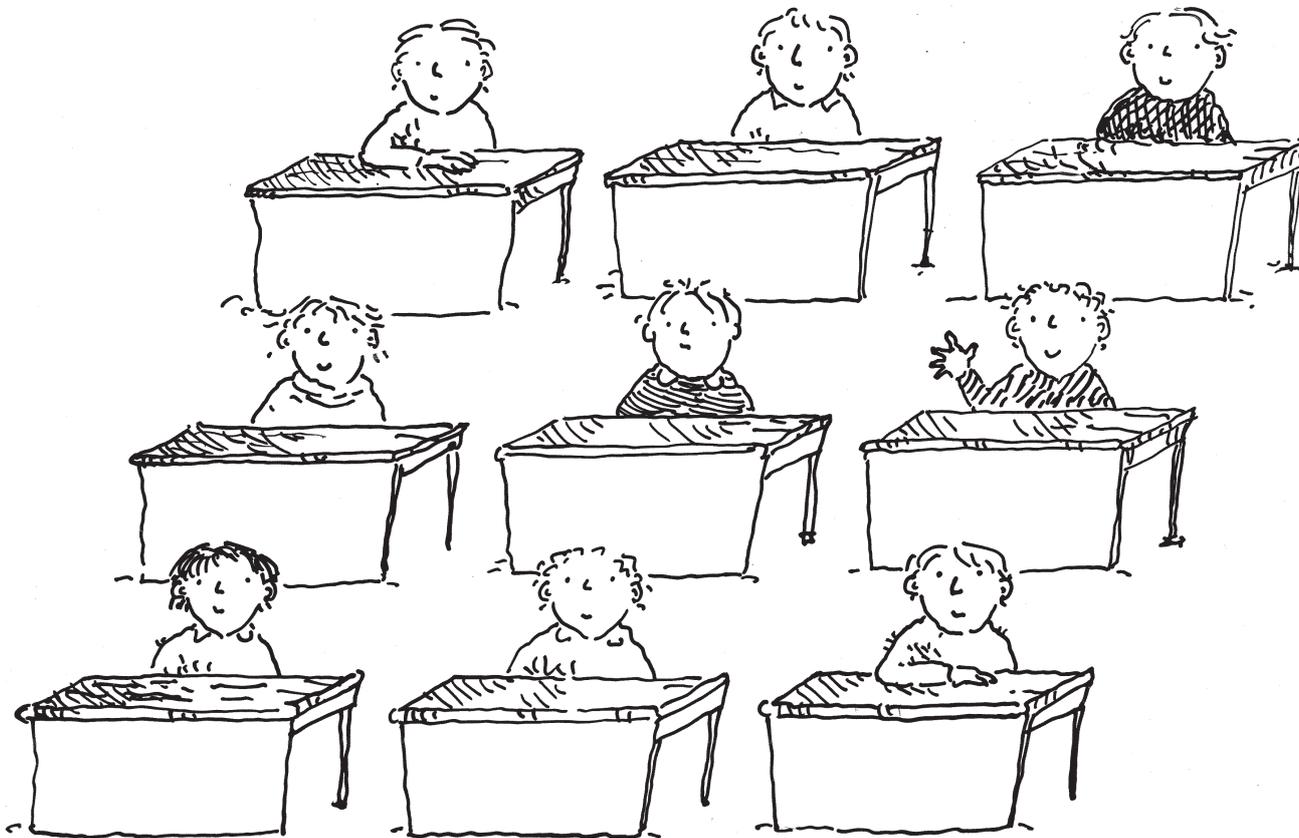
dogs _____; pigs _____; horses _____; sheep _____



Name _____

Date _____

Who sits where?



Dale sits between Nelly and Tyron and in front of Don.

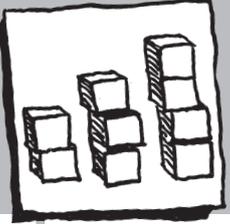
Kai sits behind Tyron.

Jana sits in front of Nelly.

Ginny sits between Jana and Aiko.

1 Where does Russ sit? _____

2 Write the children's names on their desks.



Name _____

Date _____

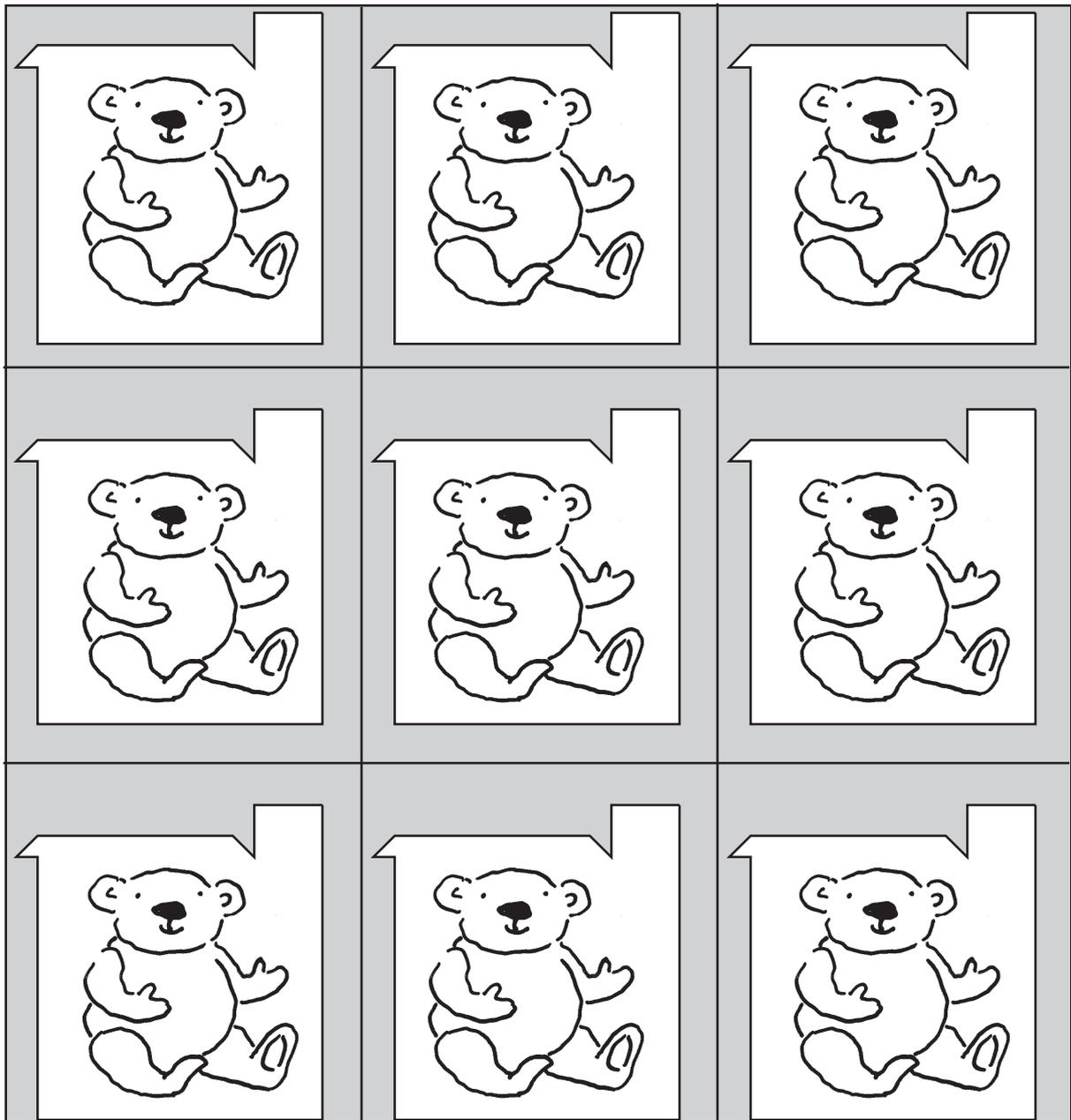
Teddy Town

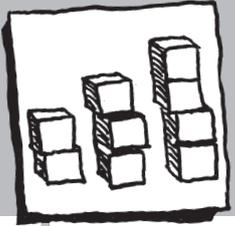
9 teddies live in Teddy Town.

3 are red, 3 are blue and 3 are yellow.

There is only one of each colour in each row and column.

Colour the teddy bears.





Name _____

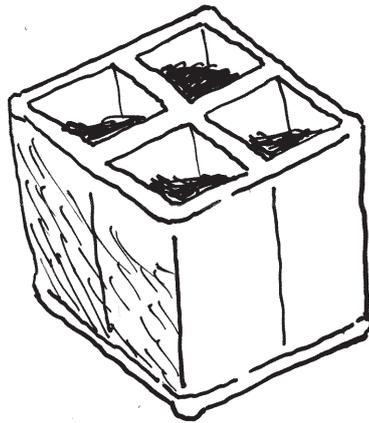
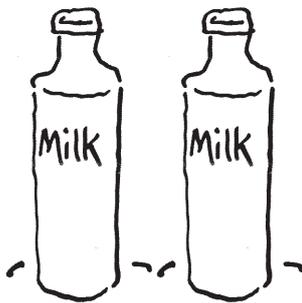
Date _____

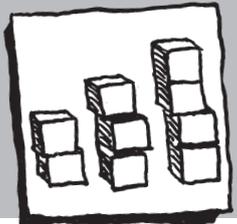
Milk bottles

Jim left out a square milk crate that could hold 4 bottles.

The milkman came with 2 bottles of milk.

Draw the different ways the milkman could put the bottles in the crate.





Name _____

Date _____

Who is left?

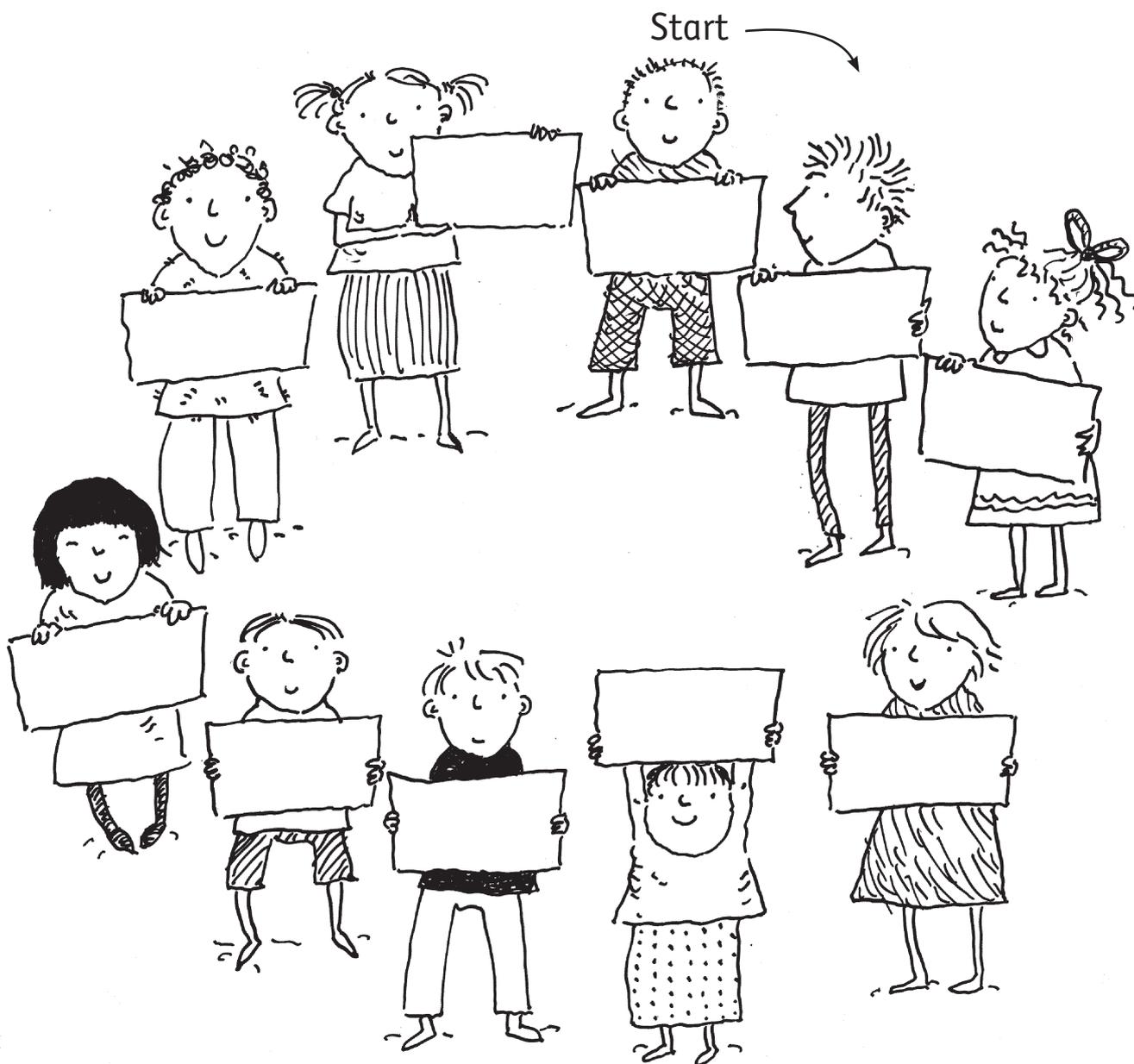
10 children are in a circle.

The children go around the circle counting to 5.

Every time they get to 5, that child sits down.

Who was out 1st, 2nd, 3rd, 4th, 5th, 6th, 7th, 8th, 9th, 10th?

Write the place onto each child.





Trial and error

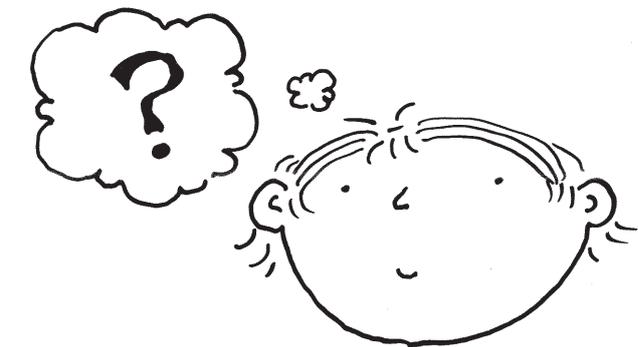
Rationale

This problem-solving strategy allows students to make guesses and then refine their guesses until they reach the correct answer. They develop logical reasoning. It also develops the understanding that problem solving can be time consuming and one has to persevere. Learning how to work systematically helps students in all their problem-solving strategies.

.....

Teaching the strategy

- Write a sample problem appropriate to your students' ages and mathematical knowledge on the board.
*eg Two numbers when added together add to 15.
The two numbers have a difference of 1.
What are the two numbers?*
- *We could solve this problem using the trial and error strategy.*
- *We first make a guess using the information in the problem. We then see how close we come to the answer. Say we guess 10 and 5. Write these two numbers on the board. Do these numbers answer the question? They add up to 15, but their difference is too big. The difference is 5 and not 1, so these numbers are not correct.*
- *We can make another guess. The numbers could be 9 and 6. These numbers add to 15, but there is still too big a difference.*
- *We have to keep guessing until we get the correct answer. We narrow the possibilities until finally we are able to find the correct answer.*
- *We can make another guess, 8 and 7. We check to see if these are the numbers. $8 + 7 = 15$ and the difference between 7 and 8 is 1. So the correct numbers are 7 and 8.*
- Discuss how wrong guesses are important steps on the way to solving a problem using this strategy.
- It is important to show working out and to be systematic in making guesses. This is essential so students can see where they are headed and also when checking the answer. Clear working can help show where errors are occurring.



As students complete a worksheet, come together as a class and have them share their solutions. Discuss how solutions were reached.

Encourage students to read the problems carefully before starting to solve them. Students make a guess, then revise and guess again until a solution is reached. Encourage them to persevere and to be systematic, as some problems will take time to solve. Stress that there is no limit to the number of guesses they are allowed to make.

.....

Worksheet 1
50 PENCE

There are a number of different solutions to this problem. Encourage multiple answers.

Vocabulary: *too much, not enough, coins, what coins?*

.....

Worksheet 2
WHAT NUMBER AM I? and POCKET MONEY

Vocabulary: *even, less, one-digit, what number?, twice, between, how much?*

.....

Worksheet 3
ADD THE SIDES

Vocabulary: *add up, total*

.....

Worksheet 4
BIRDS IN CAGES

Vocabulary: *how many?, three more than one before*

.....

Worksheet 5
EASTER EGGS

Students use trial and error to answer the problem. Encourage students to persevere as some may get frustrated with this one.

Vocabulary: *total, one more, less than, how many?*

.....

Worksheet 6
BALLS

Vocabulary: *less than, over, how many?*

.....

Worksheet 7
PUMPKINS

Tell students there is more than one answer. Encourage them to be clear in their working out and to be systematic. Remind them that one pumpkin is always 4 kg.

Vocabulary: *weigh, kg, altogether, weighs, how much?*

.....

Worksheet 8
FLOWERS

Encourage student to write more than one answer to this problem.

Vocabulary: *equal numbers, how many?*

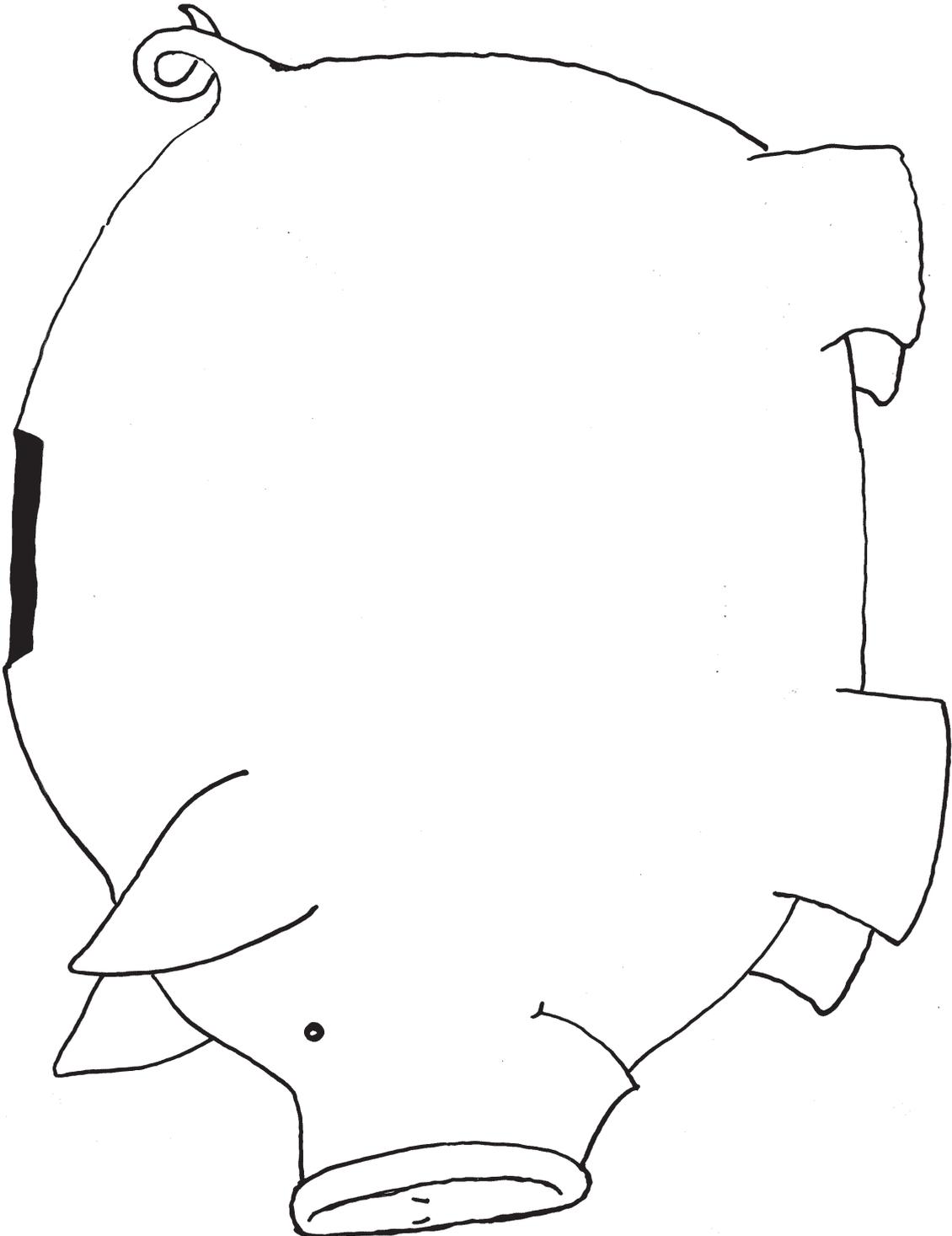


Name _____

Date _____

50 pence

Rami was given 50 pence by his dad.
What coins could Rami's dad have given him?





Name _____

Date _____

What number am I?

I am even.

I am less than 20.

I have 2 digits.

One digit is twice the other.

What number am I?



Pocket money

Reece and Luke are given £13 pocket money between them each week.

Reece gets £5 more than Luke.

How much pocket money does each get?

Reece _____ Luke _____

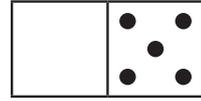
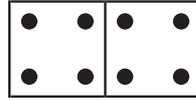
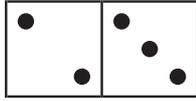
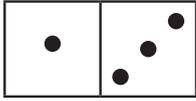




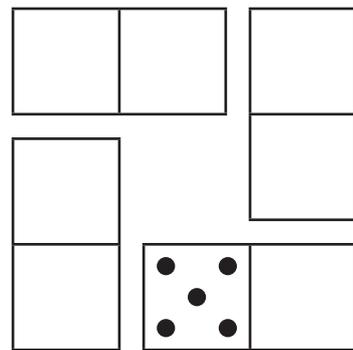
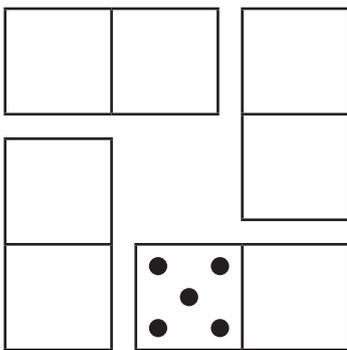
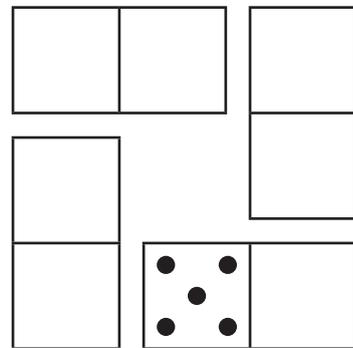
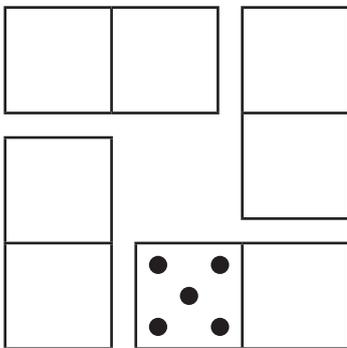
Name _____

Date _____

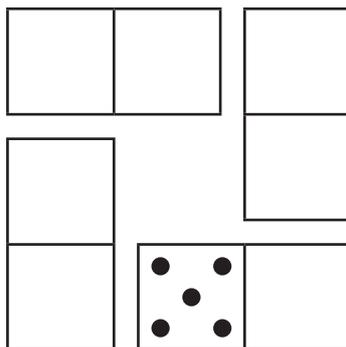
Add the sides



Finn used these dominoes to make a square.
 Each side of the square had to add to 8.
 How did he use the dominoes?



My solution:

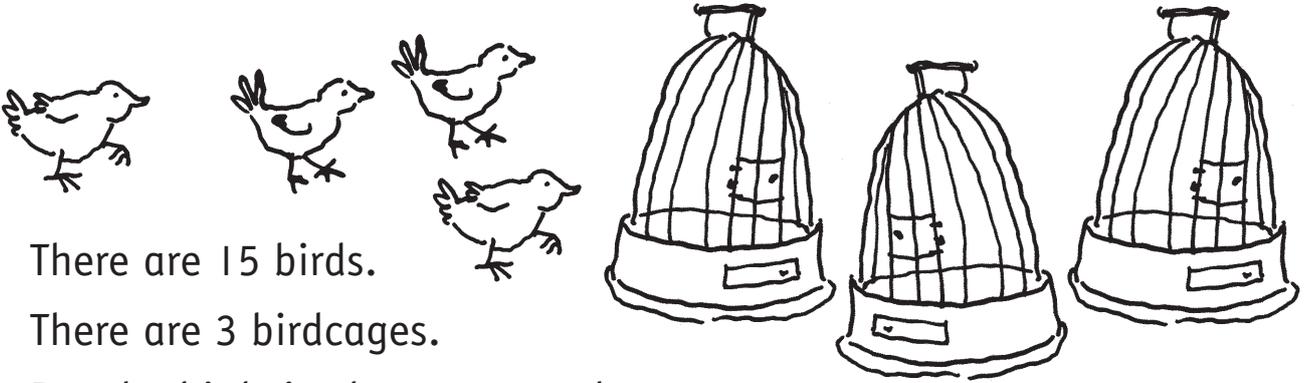




Name _____

Date _____

Birds in cages



There are 15 birds.

There are 3 birdcages.

Put the birds in the cages so that each cage has 3 more birds than the one before.

How many birds in each birdcage? _____

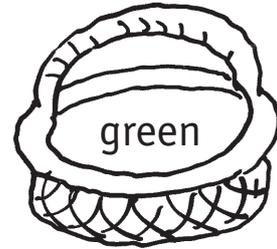
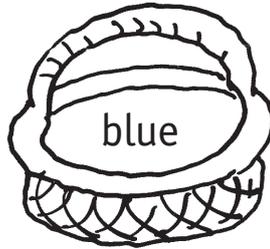
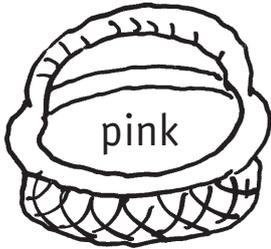


Name _____

Date _____

Easter eggs

There are 3 baskets.



There are 10 Easter eggs.

The pink basket has one more egg in it than the blue basket.

The blue basket has 3 less eggs than the green basket.

The green basket has 2 eggs more than the pink basket.

How many eggs in each basket?

pink _____ blue _____ green _____



Name _____

Date _____

Balls

The elf had less than 10 balls to pack into boxes.

If he packed the balls 2 to a box he had 1 over.

If he packed the balls 5 to a box he had 4 over.

If he packed the balls 4 to a box he had 1 over.

- 1 How many balls did he have? _____
- 2 How many balls should he pack in a box so he has none over? _____
- 3 How many boxes would he use? _____

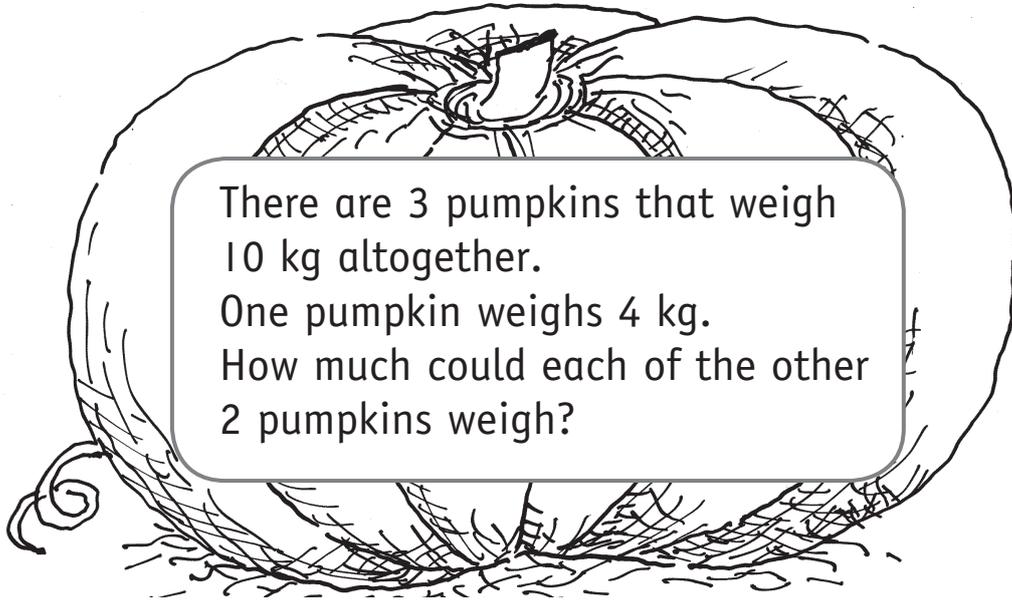




Name _____

Date _____

Pumpkins



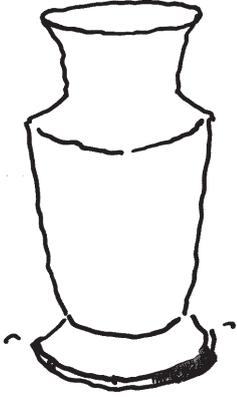
There are 3 pumpkins that weigh
10 kg altogether.
One pumpkin weighs 4 kg.
How much could each of the other
2 pumpkins weigh?



Name _____

Date _____

Flowers



Nadia has 24 flowers.
She wants to put equal numbers
of flowers into each vase.
How can Nadia place the flowers?



1 How many vases does she need? _____

2 How many flowers are there in each vase? _____



Make a list

Rationale

Learning how to structure an investigation is an important part of developing mathematical thinking. Making a list is a systematic method of organising information in rows and/or columns. This strategy allows students to clearly examine data and draw conclusions more easily than they could by just looking at unorganised numbers. They discover patterns or relationships in the data and use them to solve the problems.



Teaching the strategy

- Write a problem appropriate to your students' ages and mathematical knowledge on the board.
*eg Max raced his red, blue and yellow toy cars.
How many different ways could they finish?*

- *One way to solve this problem is to make a list.*
Write the list on the board as you speak.

1st	2nd	3rd
-----	-----	-----

- We could place the red car first.

1st	2nd	3rd
red	blue	yellow

- Then we can swap blue and yellow.
red yellow blue

- *Are there any other ways we can put the red car first? No – so then we can put the blue car first.*

blue	yellow	red
blue	red	yellow

- *Are there any other ways we can put the blue car first? No – so then we can put the yellow car first.*

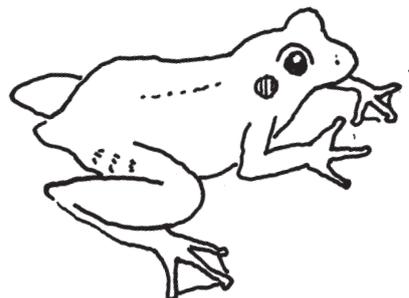
yellow	blue	red
yellow	red	blue

- *Are there any other ways we can put the yellow car first? No – so there are no more ways to put the cars.*

- *How many different ways could the cars have finished? Point out to the students how the answer can be 'seen'. 6.*
- Read the question again. *Have we answered the question? Yes we have.*
- Look at the setting out of the list, reinforcing that if students are systematic they will have a list of all possibilities and avoid repeating themselves.

As worksheets are completed ask some students to share their solutions with the class.

Encourage students to find similar problems or to write their own problems for the class or for small groups to solve. Problems can be collected and made into class books for students to share.



Remind students that they are using the *make a list* strategy to answer the problems. Remind them also to be systematic in their working out.

Worksheet 1
ASH'S BLOCKS

Vocabulary: *different, combinations*

Worksheet 2
THE DOG SHOW

Vocabulary: *different, combinations*

Worksheet 3
DARTS

Tell students that they can use a number more than once in a combination and to make sure that the scores are different in their final answer.

$10 + 2 + 5 + 8 = 25$ is the same as
 $5 + 5 + 5 + 5 + 5 = 25$.

Vocabulary: *lowest, highest, different, combinations, same*

Worksheet 4
MAKING NUMBERS

Tell students that they can make 1-, 2-, or 3-digit numbers.

Vocabulary: *3-digit numbers, largest, smallest*

Worksheet 5
PASS THE PARCEL

Vocabulary: *different, combinations*

Worksheet 6
FROGS

Tell students that 2 frogs and 6 frogs is not the same as 6 frogs and 2 frogs.

Vocabulary: *different, combinations, not the same as*

Worksheet 7
RUBY'S ICE-CREAM

Vocabulary: *different, combinations*

Worksheet 8
DICE THROW

Students can work in pairs. They need 2 dice. Tell students that $2 + 3 = 5$ is different to $3 + 2 = 5$.



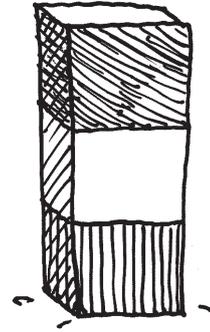
Name _____

Date _____

Ash's blocks

Ash had three blocks: a red block, a yellow block and a blue block.

How many different towers could he make using the three blocks?





Name _____

Date _____

The dog show

*Bella**Sherlock**Jango**Taz*

Four dogs are in the dog show.

How many different ways can they finish 1st, 2nd, 3rd and 4th?



Name _____

Date _____

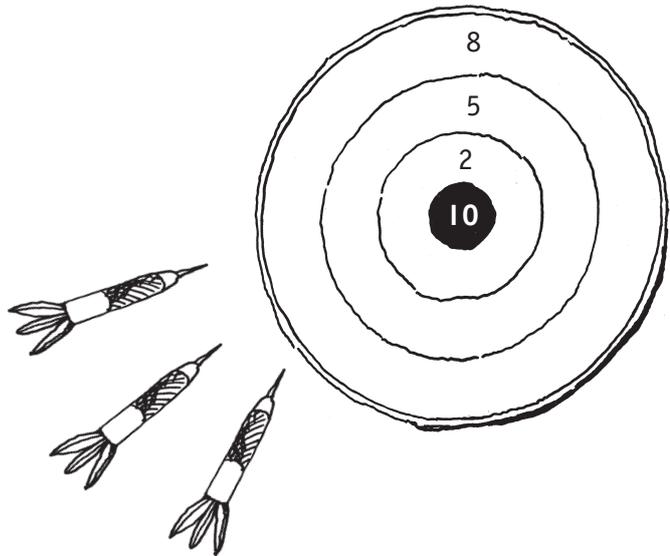
Darts

Dani throws 3 darts at the dartboard.

What scores can she make?

What could be the lowest score? _____

What could be the highest score? _____





Name _____

Date _____

Making numbers

1 a What 3-digit numbers can be made with:

6

9

3



b What is the largest number? _____

c What is the smallest number? _____

2 a What numbers can be made with:

5

7

1

b What is the largest number? _____

c What is the smallest number? _____



Name _____

Date _____

Pass the Parcel

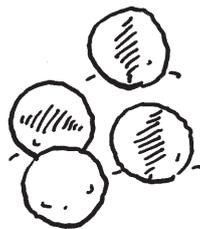
At Jason's party the children played *Pass the Parcel*.

Things in the parcel

pencil



sharpener



marbles



notebook

In how many different ways could the four things have been put into the parcel?



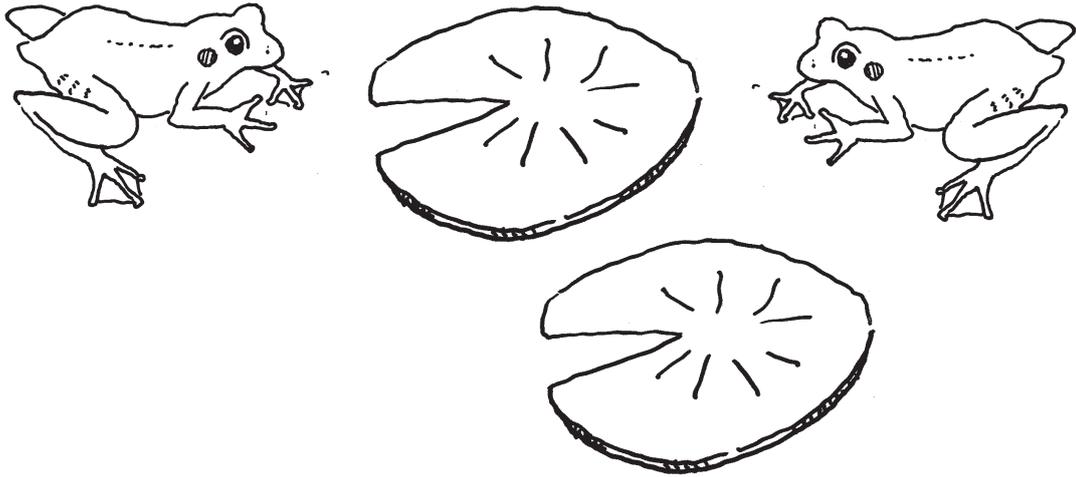
Name _____

Date _____

Frogs

There are 10 frogs and 2 lily pads.

How many ways can you arrange the frogs on the lily pads?





Name _____

Date _____

Ruby's ice-cream



Ruby wants to buy a 3-scoop ice-cream.
In what order can she have the scoops?

pink**yellow****green****blue**

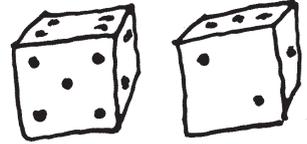


Name _____

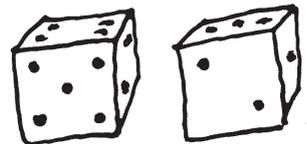
Date _____

Dice throw

- 1 Fran threw two dice.
The numbers added to 7.
What numbers could she have thrown?



- 2 Fran made a list of all the numbers that
could be made by adding the dots on two dice.
She found that one number could be made
more than any other number.



What is the number? _____



Estimation

Rationale

The skill of estimation helps students in all work in mathematics. At this level, they are encouraged to develop the strategy of 'eyeballing' a small group and using this benchmark to estimate how many are in a larger group. Recognition of patterns in a collection becomes easier. When checking estimates, students are able to model equal groups. They also develop skills in counting by five's and ten's using skip counting. This helps students deal with larger numbers as their skills progress.

Teaching the strategy

It is important to develop an understanding in students that estimation means that you do not count until later. They must understand that they are not 'wrong' if they get the 'wrong' answer. Do not emphasise 'right' guesses. The idea behind estimating is to develop the skill of close guessing first rather than counting first. While students should be encouraged to check to see if their answer makes sense in relation to the problem, they need not be required to confirm an estimate by counting. However, you will probably find that they will want to do this anyway.

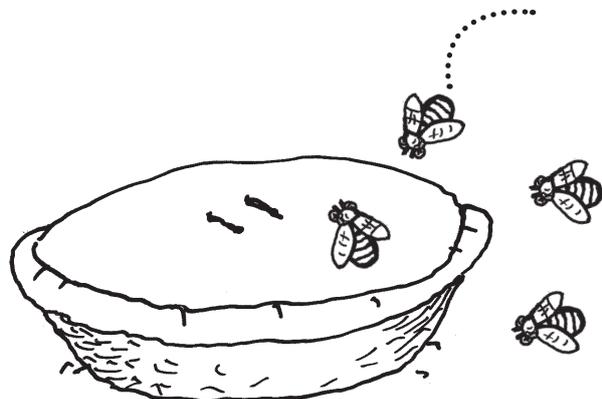
- Model how a benchmark can be used to help students estimate a larger group.
- Put out a group of five objects, eg books, school bags or marbles. Have students 'see' that it is a group of five.
- Put out a larger group of objects, eg 20, and have students estimate how many in the group.
- To check the guesses show how the objects can be put into groups of five and then the groups can be counted by 5s.
- Discuss how much easier this is than counting by ones, especially when big numbers are involved.

- Discuss when, if counting a large number of items, miscounting occurs, it is much easier to recount in groups of ten rather than by one's.
- Always encourage students to talk about finding and counting in 'groups of'.

Use opportunities that occur naturally though the day to develop estimation skills, and encourage students to recognise patterns in a collection of objects.

To enable students to complete some of the worksheets, model how to circle groups of five and ten. Model how they can then count by 5s or 10s. Discuss how this is much easier than counting by ones. As a class, regularly practise this skill.

As students complete a worksheet discuss it as a class.



Students use the estimation strategy for these problems. Stress that they do not count first. They check their guesses where possible by making groups and then counting in 5s or 10s.

Worksheet 1
FLY PICNIC

Students 'eyeball' 5 flies before estimating.

Vocabulary: *how many?, estimate, guess, don't count yet, check, groups of five, count in 5s*

Worksheet 2
FISH

Students 'eyeball' 5 fish before estimating.

Vocabulary: *how many?, estimate, guess, don't count yet, check, groups of five, count in 5s, count and check*

Worksheet 3
BOOKS

Students 'eyeball' 10 books before estimating.

Vocabulary: *how many?, estimate, guess, don't count yet, check, groups of 10, count in 10s*

Worksheet 4
AUNT JEN'S FLOWERS

Students 'eyeball' 10 flowers before estimating.

Vocabulary: *how many?, estimate, guess, don't count yet, count to check, groups of 10, count in 10s*

Worksheet 5
WHERE ARE THE SHEEP?

Students 'eyeball' 10 sheep and then guess which paddock has 90 sheep in it. Students then group sheep in groups of 10.

Vocabulary: *estimate, guess, don't count yet, check, groups of 10, count in 10s, paddock*

Worksheet 6
PETER'S TREES

Students 'eyeball' 10 trees and then guess how many more trees are needed. They guess again for dead trees.

Vocabulary: *how many more?, estimate, guess, don't count yet, check, groups of 10, paddock, plant, replant, 100*

Worksheet 7
CENTURY TOWN

Students 'eyeball' 10 houses before estimating. They observe 100 houses before further estimation. There is a challenge.

Vocabulary: *how many more?, estimate, guess, don't count yet, check, groups of 10, count in 10s*

Worksheet 8
LEROY'S TOYSHOP

Students 'eyeball' a box of 25 boats then estimate missing items.

Vocabulary: *how many?, estimate, guess, don't count yet, check, count*



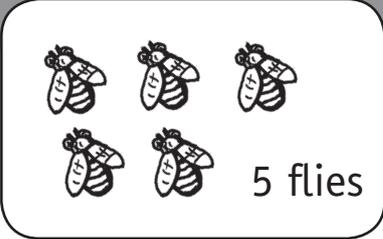
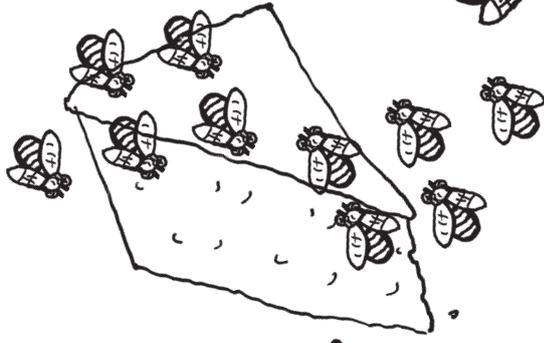
Name _____

Date _____

Fly picnic

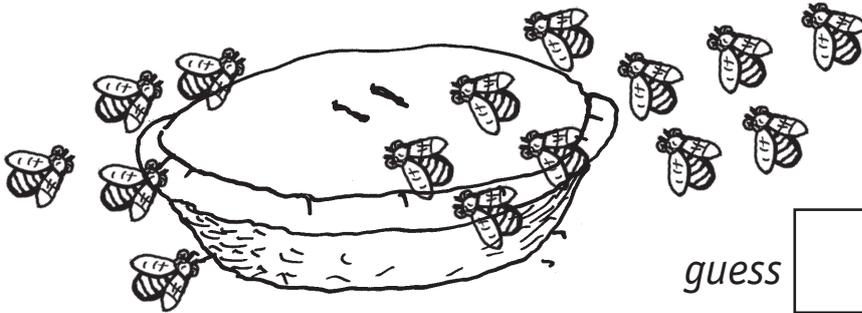
Guess how many.

1



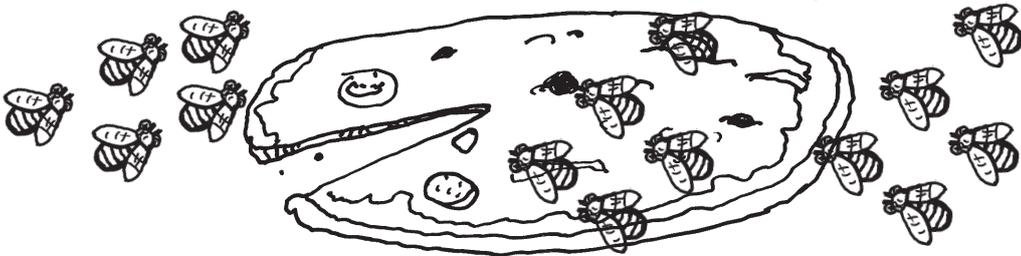
guess check

2



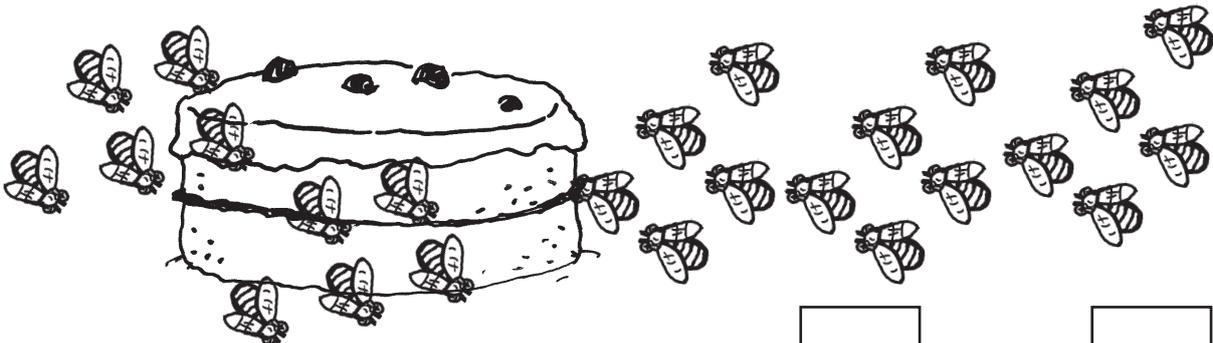
guess check

3



guess check

4



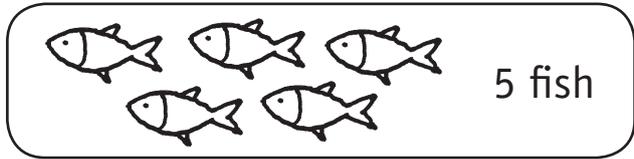
guess check



Name _____

Date _____

Fish



1 Guess how many fish. Colour your guess.

10

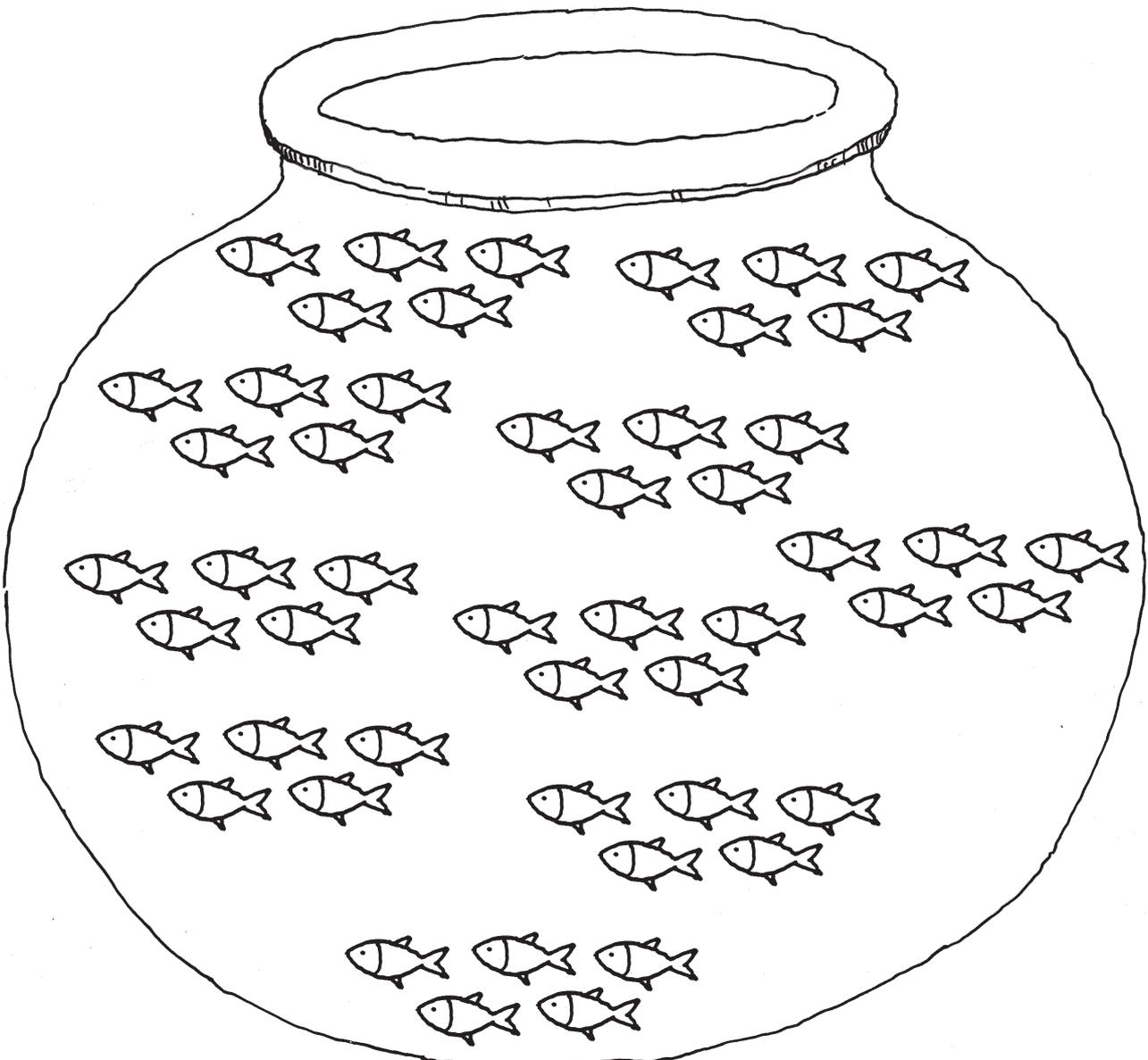
45

55

30

50

65



2 Count and check.

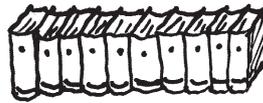
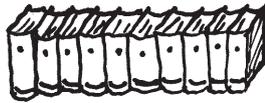
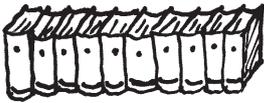
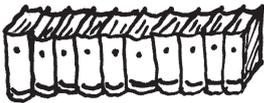
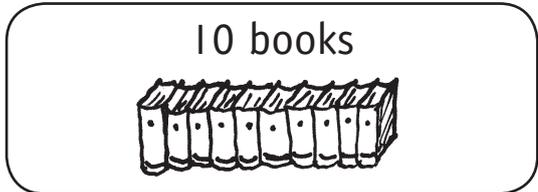


Name _____

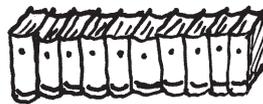
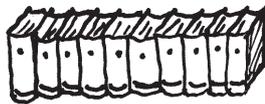
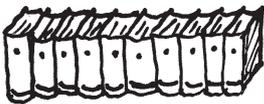
Date _____

Books

1 How many books did Sid read in the Read-a-thon?

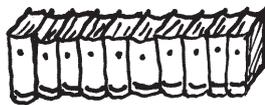
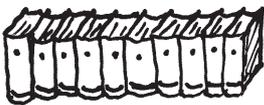


guess



count

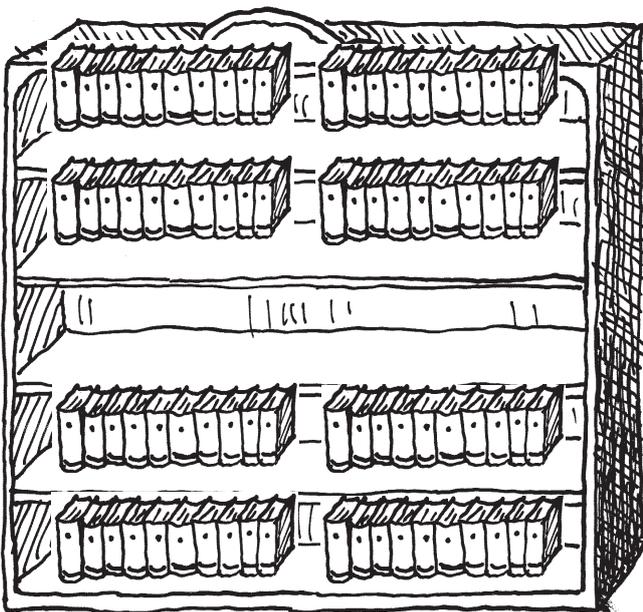
2 Bree wanted to read 50 books. How many more did she have to read?



guess



count



3 Jett had room for 100 books. How many are missing?

guess

count

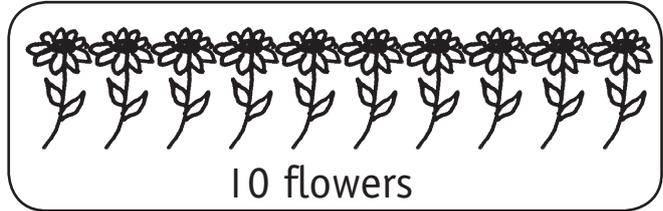


Name

Date

Aunt Jen's flowers

1 Guess how many flowers.
Colour your guess.



50

80

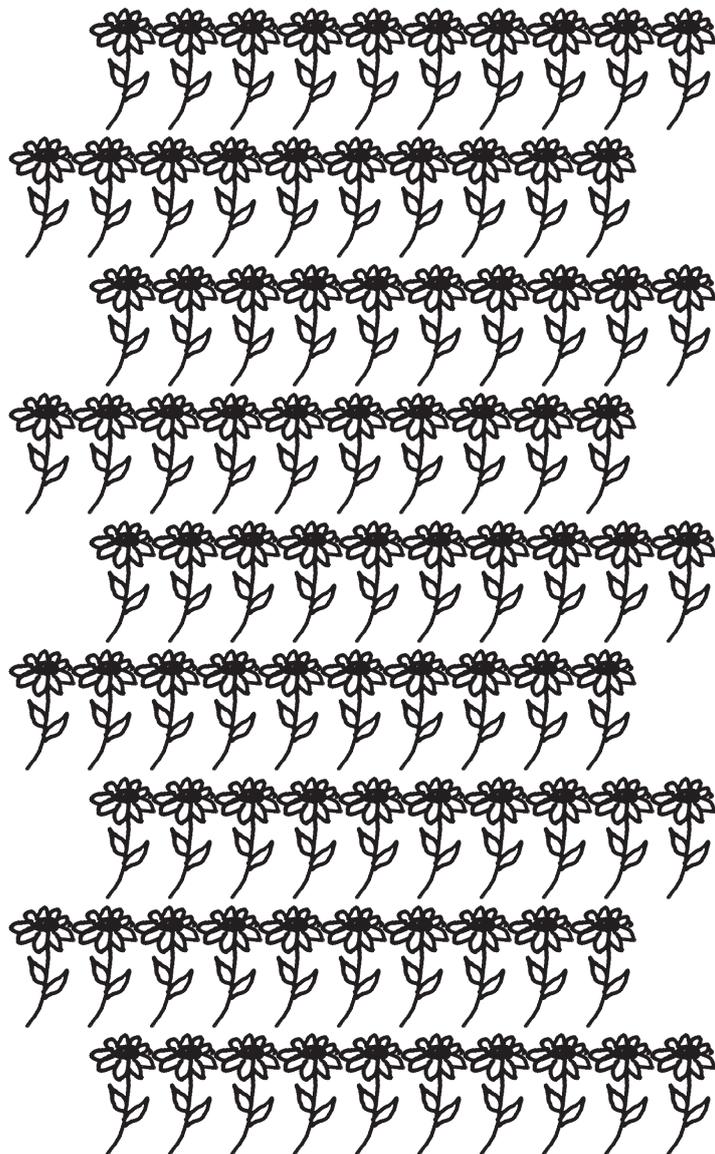
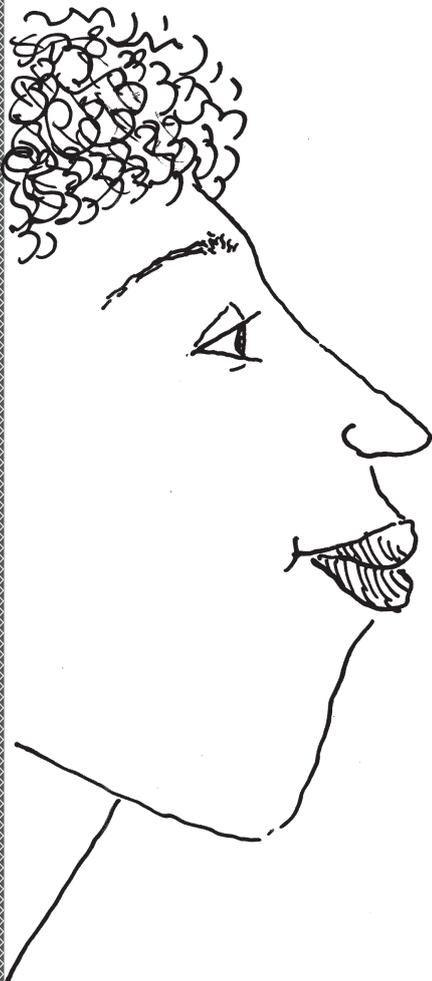
100

60

70

90

40



2 Count to check.



Name _____

Date _____

Where are the sheep?

Farmer Brown cannot remember in which paddock he put 90 sheep.
Help him to find them.

10 sheep

paddock 1

paddock 2

paddock 3

paddock 4

paddock 5

paddock 6

guess paddock

check paddock

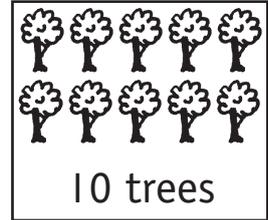
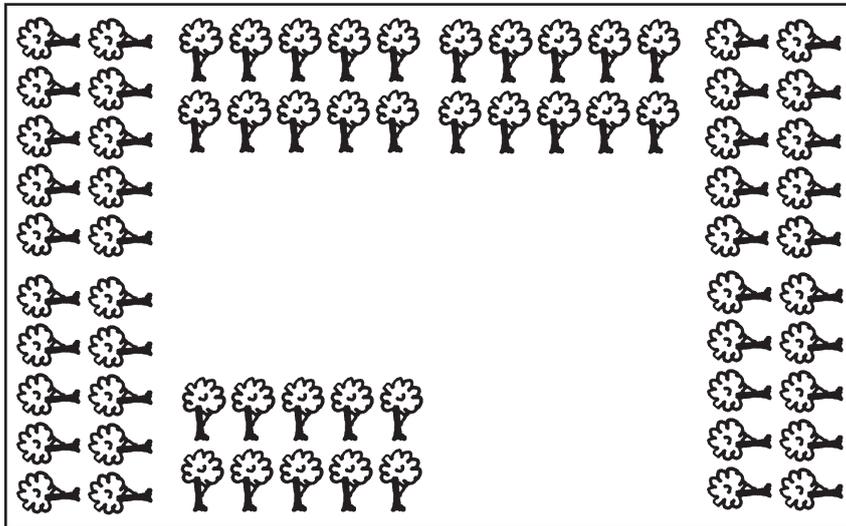


Name

Date

Peter's trees

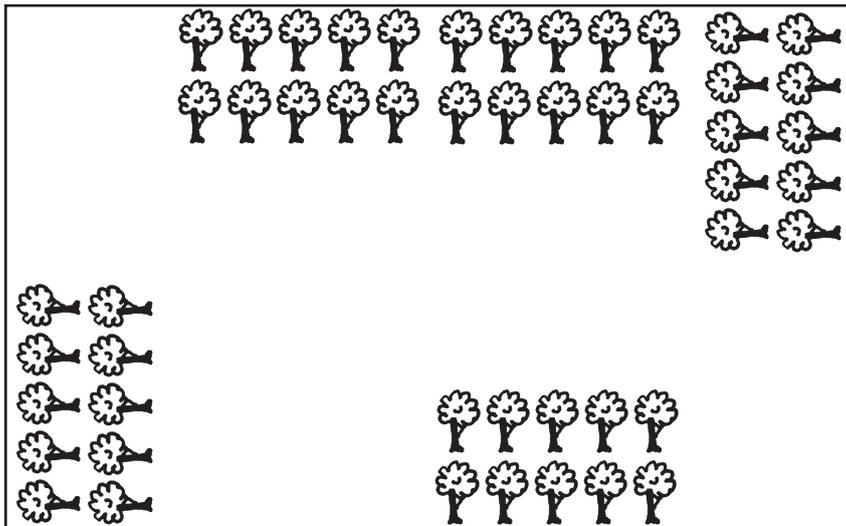
1 Peter has 100 trees to plant.



How many more trees does he have to plant?

guess check

2 Some of the trees die.



How many trees does Peter have to replant?

guess check



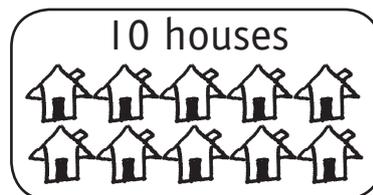
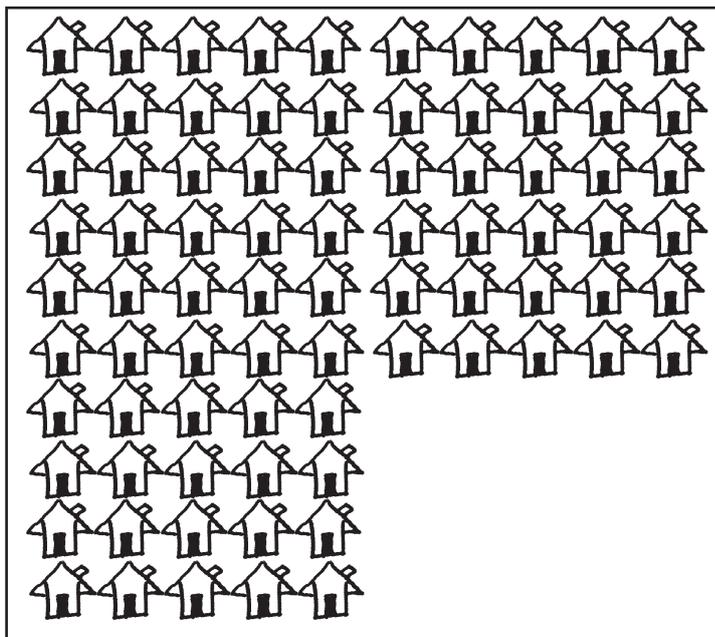
Name _____

Date _____

Century Town

Glen the builder builds 100 houses in Century Town.

1 How many more houses does he have to build?

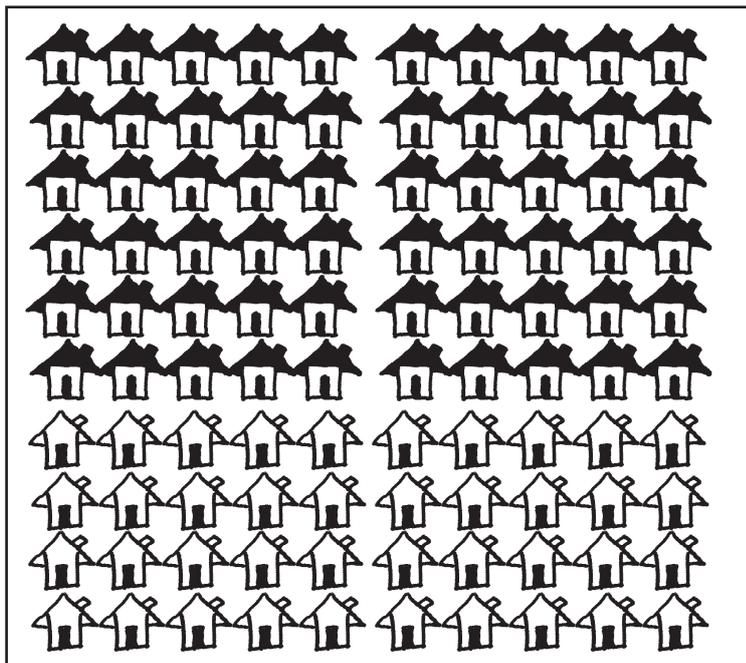


guess

count

Glen has _____ more houses to build.

2 Ubal painted the roofs of some houses in Century Town. How many houses did he paint?



guess

count

Ubal painted _____ houses.

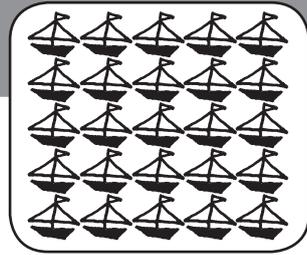
3 Challenge How many houses were not painted by Ubal? _____



Name _____

Date _____

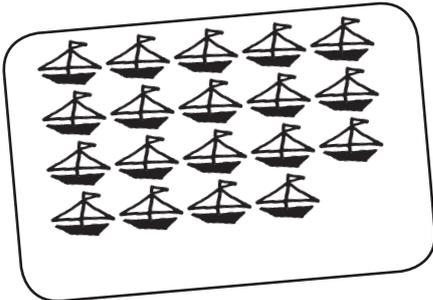
Leroy's toyshop



box of
25 boats

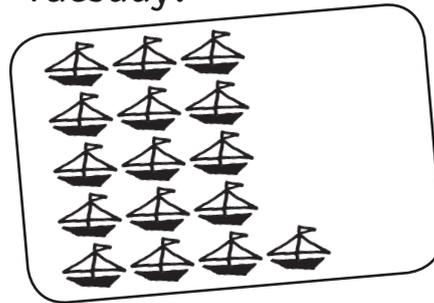
Leroy sold toy boats in his toyshop.
How many did Leroy sell on:

1 Monday?



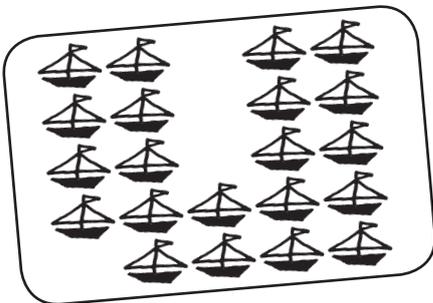
guess check

2 Tuesday?



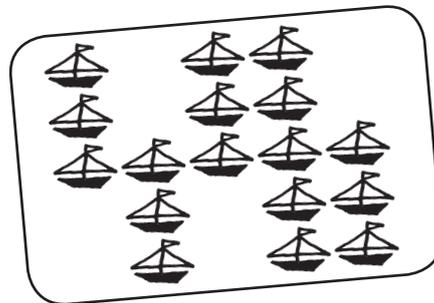
guess check

3 Wednesday?



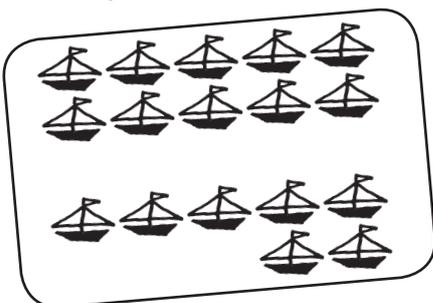
guess check

4 Thursday?



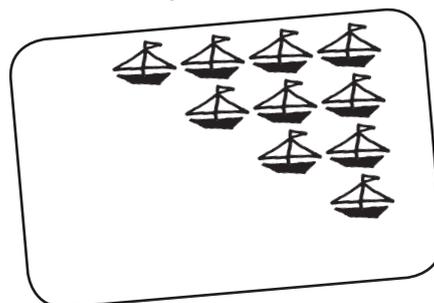
guess check

5 Friday?



guess check

6 Saturday?



guess check



Work backwards

Rationale

This strategy enables students to solve problems which do not have a 'beginning'. They begin with the final answer and work backwards through the steps. Students learn about reverse operations in order to find the data at the beginning of the problem. They choose and use appropriate operations to solve the problem.

.....

Teaching the strategy

- Write a sample problem appropriate to your students' ages and mathematical knowledge on the board.

eg Moe made some cakes for the school fete. 6 cakes were bought. Moe gave 2 cakes to Joan to take home. She had 4 cakes left over. How many cakes did Moe bake?

- Read the problem carefully with the students. *What information have we been given? We know that Moe sold 6 cakes, gave 2 cakes to Joan and had 4 cakes left over. What don't we know? We don't know how many cakes Moe baked.*
- *We can use a working backwards strategy to solve the problem.*
- Model the strategy. *In this strategy we have to work backwards.*
- *Firstly, we have to start with the information we are given at the end of the problem.*
4 (the cakes Moe has left).
- *Now we have to find all the cakes that were bought or taken.* Have the students help.
 $6 + 2 = 8$
- *Now we have to add these two answers together,*
 $4 + 8 = 12$. *Therefore Moe must have baked 12 cakes.*
- *How could we check to see if we are right? Moe baked 12 cakes. She sold 6 ($12 - 6 = 6$), she gave 2 away ($6 - 2 = 4$) and that is the 4 that Moe had left so we are correct.*

- As this strategy can be quite tricky it is important to model lots of similar problems for students, so they grasp the strategy and understand how it works.

When students do the worksheets encourage them to write down all their working and to present it clearly so they can keep a check of where they are going. This helps them when they are checking their work. Ask some students to share their solutions with the class. Discuss how they arrived at their solution.

For further practice look at problems in student workbooks as a class and discuss what operations are needed to solve them. Encourage students to find similar problems or to write their own for the class or for small groups to solve. Problems can be collected and made into class books.



Students will use the *work backwards* strategy for these problems. Make sure that they read each problem carefully so they know exactly what they have to find. In some cases you may have to help them with a starting point.

Worksheet 1
BIRTHDAY MONEY and **FISH**

2 problems involving addition.

Vocabulary: *how much?, left, how many?*

Worksheet 2
CATHY'S CAKE and **FROGS**

1 involves subtraction; 2 involves addition.

Vocabulary: *how many?, what time?, o'clock, hours*

Worksheet 3
SNAKES and **SHELLS**

1 involves multiplication and division;
2 involves addition.

Vocabulary: *how many? multiply, divide, add*

Worksheet 4
FLOWERS and **SWEETS**

1 involves fractions; 2 involves addition and lots of five.

Vocabulary: *how many?, one-quarter, one-half, addition, lots of, 5s, as many*

Worksheet 5
HOW OLD?

Involves subtraction, halves and addition.

Vocabulary: *how old?, subtract, half, addition, add, take away, older, younger*

Worksheet 6
OCTOPUS RIDE

Involves multiplication and addition.

Vocabulary: *multiply, add*

Worksheet 7
SUE'S DAY

Involves subtraction of hours. Show students how to count backwards on a clock. Remind them to answer in *o'clock*.

Vocabulary: *what time?, before, after, left, hours, o'clock*

Worksheet 8
JELLY BEANS

Involves addition, recording and drawing.

Vocabulary: *how many?, add, total, less, more*



Name _____

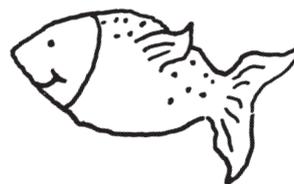
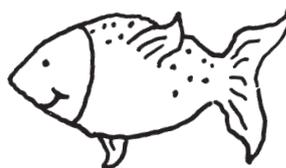
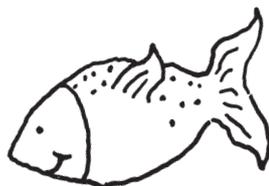
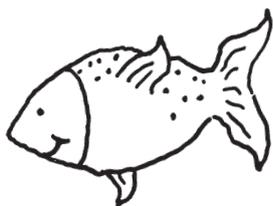
Date _____

Birthday money



Leah was given some money for her birthday.
She bought a doll for £13. She had £8 left.
How much money did she get for her birthday?

Fish



There was a school of fish.

7 swam away.

15 fish stayed.

How many fish were in the school? _____

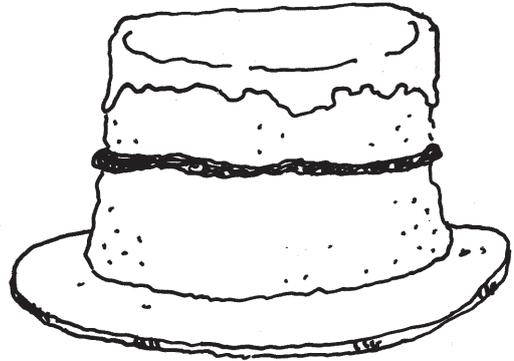


Name _____

Date _____

Cathy's cake

Cathy put the cake in the oven.
It took 2 hours to cook.
Cathy took the cake out of the
oven at 4 o'clock.
What time did she put the cake
in the oven?



Frogs



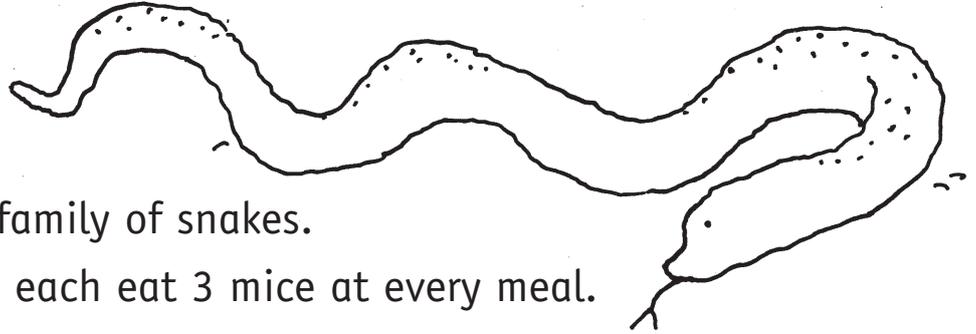
There were some frogs sitting on a log.
17 frogs jumped into the pond.
3 were left on the log.
How many frogs were sitting on the log
to start with?



Name _____

Date _____

Snakes



There is a family of snakes.

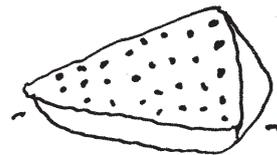
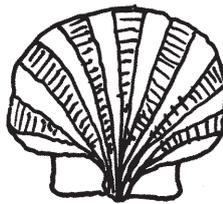
The snakes each eat 3 mice at every meal.

They eat 3 meals a day.

18 mice were eaten today.

How many snakes are in the family?

Shells



Susan collected some shells at the beach.

She gave 4 to Dane.

She gave 5 to Brooke.

If Susan had 11 shells left, how many shells did she collect at the beach?



Name _____

Date _____

Flowers

Some flowers are growing in the garden.

One-half of the flowers are red.

One-quarter of the flowers are yellow.

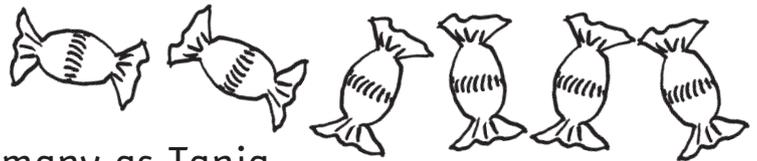
The rest are orange. 5 of the flowers are orange.

How many flowers are there?



Sweets

Tania has some sweets.



Michael had 5 times as many as Tania,

but he ate 6 and now he has 14 left.

How many sweets does Tania have?





Name _____

Date _____

How old?

Holly is 10 years younger than Rosa.

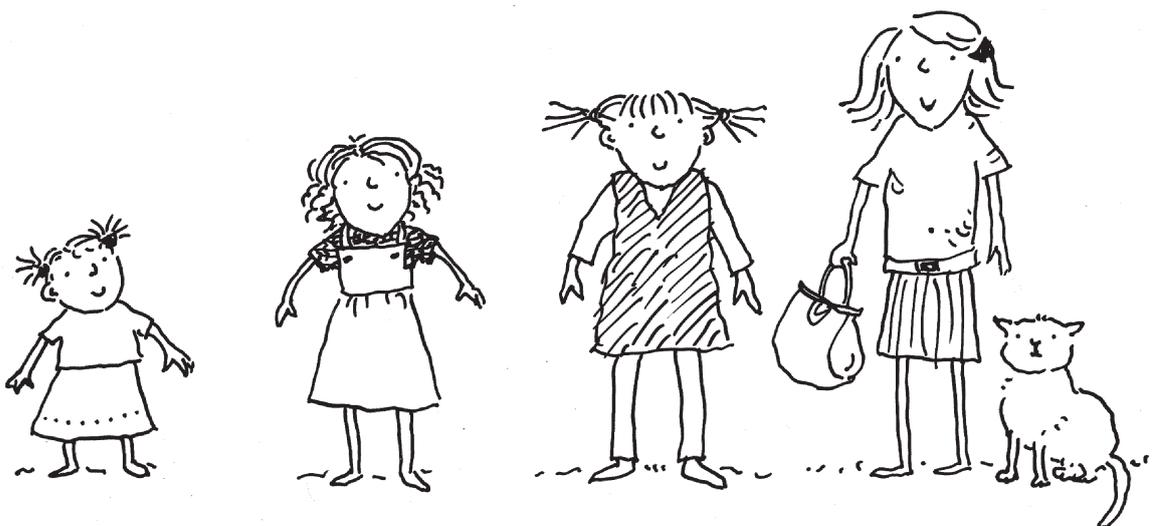
Meg is half Holly's age.

Toni is 7 years older than Meg.

Toni is 12 years old.

1 How old is Rosa? _____

2 What are their names?



3 How old are they?



Name _____

Date _____

Octopus Ride

Some children were in a line to go on the Octopus Ride.

Each cage could take 4 children.

There were 5 cages.

All the children except 3 got on the ride.

How many children were in the line?





Name _____

Date _____

Sue's day

- 1 Sue ate breakfast 1 hour after she got out of bed.
She ate breakfast at 8 o'clock.
What time did she get up? _____



- 2 Sue went to school.
She was there for 6 hours.
She left school at 3 o'clock.
What time did she start school? _____



- 3 After school she played, had a bath, then had dinner.
She ate dinner 2 hours before she went to bed.
She went to bed at 8 o'clock.
What time did she eat dinner? _____





Name _____

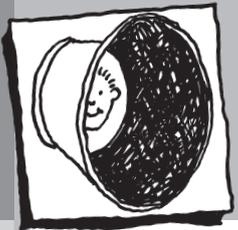
Date _____

Jelly beans



- 1 Tom has some jelly beans.
There are 6 yellow jelly beans.
There are 6 less orange than green jelly beans.
There are 4 more green jelly beans than red jelly beans.
There are 4 red jelly beans.
How many jelly beans altogether?

- 2 Draw and colour the jelly beans.



Open-ended

Rationale

Open-ended problems cater to a wide range of mathematical abilities and stages of development in students. Students can feel 'successful' finding just one possible solution while more able students can give a systematic presentation and explanation of every possible solution. Learning how to work systematically helps students with all their problem-solving skills.

Teaching the strategy

- Write a problem appropriate to your students' ages and mathematical knowledge on the board.
eg You have £10 to spend. How could you spend it and have no change?
 - Draw and label some items, eg ball £4, car £3, doll £6, pencil £1, book £5.
 - As with all problem solving encourage students to read the question carefully and decide what they are asked to find.
 - Ask a student for a possible solution and write it on the board. Ask another student for a possible solution and write it on the board.
 - *Problems like these have more than one answer so they are called open-ended problems.*
 - *Where would be a good place to start? What is the best way to record the answers? What strategies can we use to make sure we get all the combinations?*
 - Stress how important it is to be methodical and to show working. By doing this students can keep check of where they are going in the solution.
 - Ask students for other possible solutions.
 - Model how the combinations can be written so there is a systematic list.
 - Ask students for a few more combinations and write them on the board.
- *Have we answered the question? Yes, we have, however some of you may be able to think of more combinations. There are many correct answers.*

When students finish a worksheet ask some to share their solutions and encourage them to explain their thinking orally. This allows them to clarify their own ideas and to reflect critically on their work. Discussion and sharing will also develop understanding that there are many ways to reach a solution.

Encourage students to write their own problems for the class or for small groups to solve. Problems can be collected and made into class books for students to share.



All these problems are *open-ended*. When students are completing the worksheets remind them that there is more than one solution for each problem. Encourage more able students to find many solutions and encourage all students to persevere when working through the problems.

Worksheet 1
TEN FISH

Remind students they can have 3 and 7, and 7 and 3 as they are different.

Vocabulary: *how could?, and, add, total, different*

Worksheet 2
£20 TO SPEND

Items can be bought more than once. Good problem for discussing solutions.

Vocabulary: *add, total, addition, equals, no change*

Worksheet 3
HUNGRY GILBERT SNAIL

Remind students how to find half of a collection. Good problem for discussing solutions.

Vocabulary: *half, two equal parts, grid, cross out*

Worksheet 4
FIVETOWN

Tell students that at least one full side of each square has to be touching another full side to join the squares. Remind them that all drawings have to be different.

Vocabulary: *different, five, on top of, side of, up, down, across*

Worksheet 5
EGG HATCH

Students could work in pairs or small groups. A 100 chart can be used to help find the next numbers in the pattern. More able students can go further than 100.

Vocabulary: *number pattern, how many?*

Worksheet 6
BOOK BALANCE

Tell students they can use two or more piles together as long as each pile is used just once in each combination.

Vocabulary: *balance, pile, add, total*

Worksheet 7
TOM THE PACKER

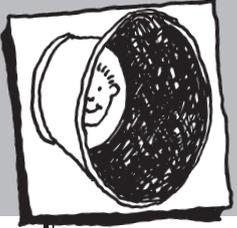
Students could work in small groups. Perhaps model one solution, eg 100 in one box stacked in rows of 10. Good problem for discussing solutions.

Vocabulary: *stack, how many?, 100, full, same, rows, deep, high*

Worksheet 8
PETS

Students can work in small groups. Model drawing legs using strokes to ensure that there are not more than 12 legs. Tell students that they can have multiples of any pet.

Vocabulary: *total, combinations, 4s, 2s, 6s, eight*



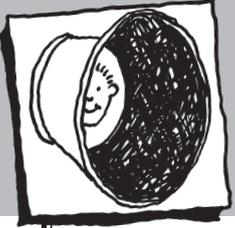
Name _____

Date _____

Ten fish

There are 10 fish and 2 fish bowls.
How can the fish be put into the fish bowls?





Name _____

Date _____

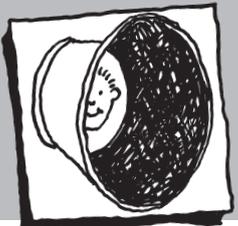
£20 to spend

Maria went to the shop.

She had to spend £20 and have no change.

What could she have bought?





Name _____

Date _____

Hungry Gilbert Snail

Julian planted 16 lettuces.

Gilbert Snail was so hungry he ate half of the lettuces in Julian's garden.



Show by  the different ways that Gilbert could eat half of the lettuces.

1

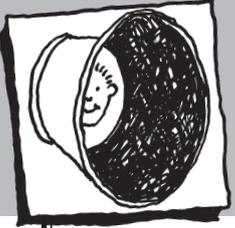
2

3

4

5

6



Name _____

Date _____

Egg hatch

Farmer Brown's hens laid some eggs.



- 1 chicken hatched on Monday.
- 3 chickens hatched on Tuesday.
- 6 chickens hatched on Wednesday.
- 10 chickens hatched on Thursday.
- 15 chickens hatched on Friday.



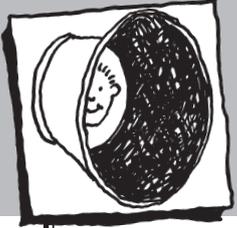
- 1 How many chickens hatched on Saturday?

_____ chickens hatched on Saturday.

- 2 How many chickens hatched on Sunday?

_____ chickens hatched on Sunday.

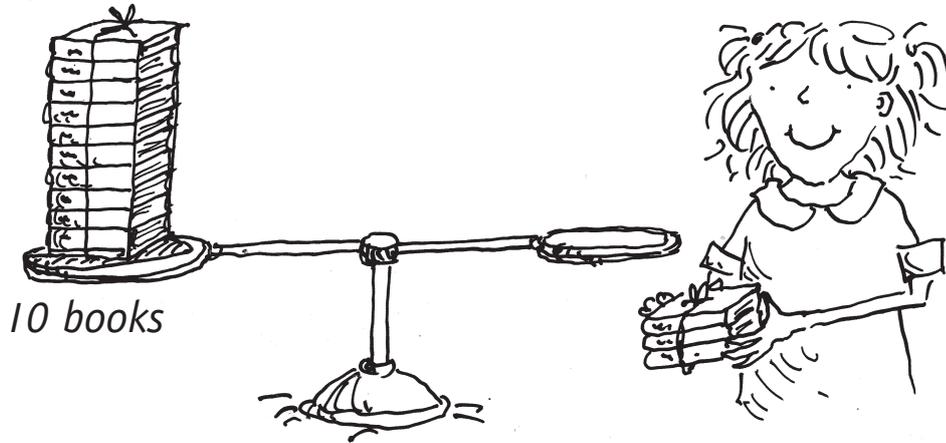
- 3 Keep following the pattern as far as you can.



Name _____

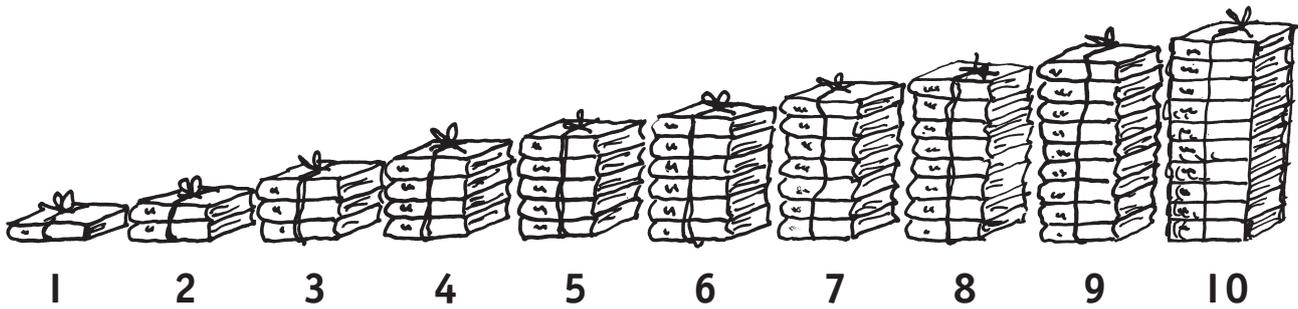
Date _____

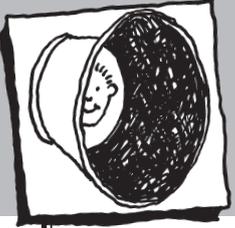
Book balance



Which piles of books could be used to balance the ten books?

(**Note:** All the books are the same weight.)





Name _____

Date _____

Tom the packer

Tom the packer has 100 juices to pack into cartons.

Each carton has to be full.

- 1 In how many ways can he stack the juice into cartons?

- 2 How many cartons will he need each time?





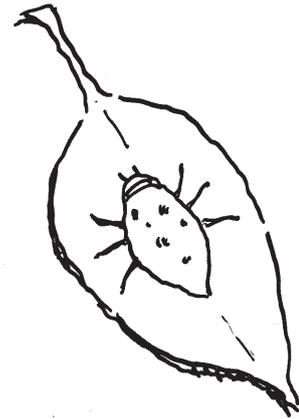
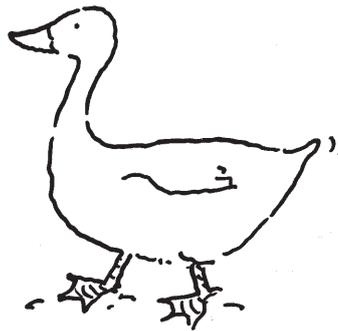
Name _____

Date _____

Pets

Work with a friend or in a small group.

Leah has some of these pets.



Altogether the pets have 12 legs.

What pets could she have?